Analysis of the Solidification Process of Castings Depending on Their Configuration and Material of the Mold

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Control of the formation of the structure and specified properties of castings in foundry production processes is inextricably linked to the thermal conditions of solidification of the castings in the mold. The nature of the thermal interaction between the casting and the mold is largely determined by the configuration of the castings as well as the properties of the cast alloy and mold material. The analysis performed in this work shows that the numerical models and empirical formulas used to calculate casting solidification parameters can be divided into three groups. The first group of models and empirical formulas gives values approaching the results of calculations by the square root law. The second group includes models and formulas, the calculation of solidified skin thickness by which exceeds the results of calculations by the square root law. The third group includes models and empirical formulas, which provide calculated data close to the theoretical curves of solidification of classical bodies. According to the results of the analysis of calculated data on the basis of the considered models, a hypothetical mechanism of the solidification process of castings has been proposed, which explains the stages of formation of their structure and the nature of the deviation of experimentally obtained values of solidification parameters from the square root law.

Keywords: casting, alloy, solidification parameters, numerical models, empirical relationships, heat transfer, casting mold.
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Introduction

The development of theoretical and technological foundations of alloy casting processes over the past almost century period has established that the quality of casting (including alloy structure and the level of mechanical and performance properties of castings) is largely determined by the phenomena occurring during melt crystallization and solidification of castings [1–6]. The configuration of the castings and the mold material are also essential in this case [7–10].

The theory of foundry processes as a specific direction in considering the above aspects has been formed about the last 50-70 years. Russian and international researchers have devoted numerous publications to the study of the processes of melt crystallization and solidification of castings, the classical works of which should be considered V. V. Gulyaev [4], A. I. Veinik [1, 5], N. I. Chvorinov [3, 11], A. A. Ryzhikov [12], G. F. Balandin [2], R. W. Ruddle [13], M. C. Flemings [10], A. Ono [14], etc.

Many problems associated with the formation of casting defects are caused by the thermophysical conditions of casting solidification processes in the mold [15–17]. In this case, an important role is given to the total solidification time of the casting, counted from the moment of pouring molten metal into the mold to the moment of its complete solidification. The solidification time of the castings and the cooling rate during solidification directly affect the characteristics of the cast alloy structure, such as grain size and distance between dendrite branches [18]. In addition, differences in the cooling rates of different parts of the casting during solidification lead to significant differences in the resulting structure and properties. According to well-known Chvorinov’s rule, the total solidification time of the casting of a simple configuration correlates with the volume and surface area of the casting so that $T_{TS} = C_m(V/A)^2$, where T_{TS} – total solidification time, C_m – mold constant, V – volume of casting, and A – surface area of casting [19]. Among other aspects, the total solidification time of a casting is an important economic characteristic for industrial conditions, allowing to evaluate the efficiency of production processes. In this connection, considerable attention in the modern theory of foundry processes is paid to the development of approaches to the evaluation of casting solidification parameters [20].

Therefore, understanding the mechanisms of thermal interaction between the casting with the mold and the factors affecting the processes involved is important from the standpoint of casting structure control. In particular, studying the features of heat exchange between the casting and

Table 1. **Analysis of experimental data, theoretical, analytical, and numerical calculations of solidified skin thickness for castings of different configuration and mold material**

Alloy	Mold material	Object of study (casting)	Research method	Authors	Figure
1	2	3	4	5	6
		Theoretical studies			
all kinds		plate	theoretical	V. V. Gulyaev [4]	1-a
all kinds		cylinder	theoretical	V.V. Gulyaev [4]	2-a
all kinds		sphere	theoretical	V.V. Gulyaev [4]	3-a
all kinds at Bi << 1		plate casting	theoretical	A. I. Veinik [1, 5]	1-b
all kinds at Bi << 1		cylindrical casting	theoretical	A. I. Veinik [1, 5]	2-b
all kinds at Bi << 1		hollow cylindrical casting	theoretical	A. I. Veinik [1, 5]	2-c
all kinds at Bi << 1		ball casting	theoretical	A. I. Veinik [1, 5]	3-b
all kinds at Bi << 1		hollow ball casting	theoretical	A. I. Veinik [1, 5]	3-c
all kinds		plate	theoretical	G. F. Balandin [2]	1-c
all kinds		cylinder	theoretical	G. F. Balandin [2]	2-d
all kinds		ball	theoretical	G. F. Balandin [2]	3-d
all kinds		all kinds	theoretical	Y. A. Nekhendzi and N. G. Girshovich [6]	1-d, 2, 3-e
all kinds		all kinds	theoretical	$\varepsilon=k\sqrt{\tau}$	diagonal
		Metal mold			
bronze	copper	cylindrical casting ∅55 mm, water-cooled mold	vacuum suction	B. M. Ksenofontov [1]	4-a
Cr-Ni-Mo steel	cast iron	cylindrical casting ∅280 mm and weight 500 kg	thermal analysis	A. K. Zhegalov, V. M. Tageev [1]	4-b
steel	cast iron	cylindrical casting ∅1750 mm and weight 80 tons	thermal analysis	N. D. Ageev [1]	4-c
aluminum	cast iron	cylindrical casting ∅200 mm	method of pouring out	B. H. Alexander [15]	4-d
aluminum	cast iron	cylindrical casting ∅150 mm	method of pouring out	E. Marburg [15]	4-e
steel	cast iron	cylindrical casting ∅540 mm and weight 4 tons	thermal analysis	N. D. Ageev [1]	4-f
ductile iron	steel	cylindrical casting ∅120 mm	method of pouring out	N. G. Girshovich [6]	4-g
steel	cast iron	cylindrical casting ∅150 mm	method of pouring out	R. W. Ruddle [15]	4-h
rimmed steel	cast iron	casting 750×750 mm and weight 7 tons	radioactive isotopes	A. A. Zaborovsky [15]	4-i
aluminum	cast iron	plate casting 30×120×160 mm	thermal analysis	A. I. Veinik [1, 5]	2-j
aluminum	steel	plate casting 5×160×175 mm	thermal analysis	A. I. Veinik [1, 5]	4-k
aluminum +20% Zn	cast iron	plate casting 100×400×400 mm	thermal analysis	G. F. Balandin [2]	4-l
		Sand mold			
ductile iron	sand mold	ball casting ∅220 mm	method of pouring out	A. A. Ryzhikov [12]	5-a
steel	sand mold	cylindrical casting ∅400 mm	thermal analysis	N. I. Chvorinov [11]	5-b
cast iron	sand mold	ball casting ∅120 mm	thermal analysis	N. I. Chvorinov [11]	5-b
aluminum	sand mold	plate casting 120×150 mm	method of pouring out	Y. Chao, L. H. Vlack [1]	5-c
aluminum	sand mold	cylindrical casting ∅125 mm	method of pouring out	Y. Chao, L. H. Vlack [1]	5-d
cast iron	sand mold	cylindrical casting ∅80 mm	method of pouring out	O. N. Magnitsky [11]	5-e
steel	sand mold	plate casting 100×30 mm	method of pouring out	K. L. Clark [15]	5-f
steel	sand mold	ball casting ∅250 mm	method of pouring out	H. Y. Husicker [15]	5-g
steel	sand mold	cylindrical casting ∅310 mm and weight 1000 kg	thermal analysis	W. S. Pellini [15]	5-h
		Numerical simulations and empirical relationships			
steel	cast iron	cylindrical casting ∅1050 mm and weight 20 tons	numerical simulation	V. M. Golod [16]	6-a
cast iron	cast iron	the ”equivalent casting” method	numerical simulation	A. I. Veinik [1, 5]	6-b
carbon steel	cast iron	cylindrical casting ∅100 mm	empirical relationship	N. G. Girshovich, Y. A. Nekhendzi [6]	6-c
cast iron	sand mold	ball casting ∅152 mm	empirical relationship	A. A. Ryzhikov [12]	6-d
steel	cast iron	industrial ingots	empirical relationship	C. Schwarz, N. M. Lightfoot [15]	6-e
cast iron	sand mold	cylindrical casting ∅30 mm	empirical relationship	N. I. Chvorinov [11]	6-f
steel	cast iron	steel ingot	numerical simulation	L. S. Konstantinov [1]	6-g
steel	cast iron	plate casting	numerical simulation	G. P. Ivantsov [1]	6-h
cast iron	sand mold	plate casting	numerical simulation	P. G. Novikov [1]	6-i
cast iron	sand mold	cylindrical casting	empirical relationship	P. P. Berg [1]	6-j
pure iron	cast iron	cylindrical casting ∅350 mm	empirical relationship	M. C. Flemings [10]	6-k

the mold provides the basis for the development of methodological principles for estimating the rates of solidification and cooling of castings in molds made of different materials, including the prediction of the intensity of solid phase formation [21-23]. In this case, depending on the casting cooling intensity, the solidification process will be determined primarily by the conditions of heat exchange on the surface, i.e., the value of the heat transfer coefficient (at low or medium intensity), or the thermal resistance of the casting material, which depends on its thermal conductivity (at high cooling intensity) [24-27]. Under simplified boundary conditions of solidification, the growth of solidified metal skin from the outer surface into the casting can be described by the well-known square root law [28, 29]; however, to bring the results of calculations closer to the experimental data, various qualifying and correcting empirical coefficients must be introduced in addition.

In spite of the above, a number of issues remain insufficiently investigated, first of all, related to the evaluation of the efficiency of methods for studying solidification processes, which allow obtaining reproducible and accurate results [30-32]. In the first place, this concerns the significant differences between the theoretical and experimental values of casting solidification parameters observed in most cases, which requires further development of methods for their prediction.

The purpose of this work is to analyze the results of studies of the solidification process of castings from different alloys, taking into account their configuration and the material of the casting mold, to compare the theoretical and analytical calculations of the main solidification parameters, and to identify the comparative effectiveness of the methods.

Raw data for the analysis

To perform a comparative analysis, this research uses the generally accepted geometric simplifications of the casting configuration to classical bodies ("plate", "cylinder", "sphere"). This approach is motivated by the fact that in the theory of casting formation from the standpoint of thermal interaction with the mold during solidification and cooling, the entire geometric variety of castings is usually reduced to the three above groups depending on the extent in the three directions of coordinate axes (x, y, z) [33—35]. In this case, we compared the results of estimates of casting solidification parameters in gravity casting in sand and metal molds, since these types of casting are the most common in industry. In addition, published sources provide data on the solidification parameters of castings and the corresponding thermophysical characteristics of mold materials only for sand and metal molds.

Table 1 summarizes the experimental results available in scientific sources, obtained by various scientists in the field of alloy solidification processes. Theoretical and empirical dependencies are analyzed, considering the type of casting alloy, casting mold material, object (casting configuration), and research method. Each specific variant in Table 1 corresponds to the curve in Fig. 1 to 6. The curves, depending on the corresponding criteria, were combined by the authors of this work into blocks comparing the results of theoretical, analytical, and numerical calculations of the solidified skin

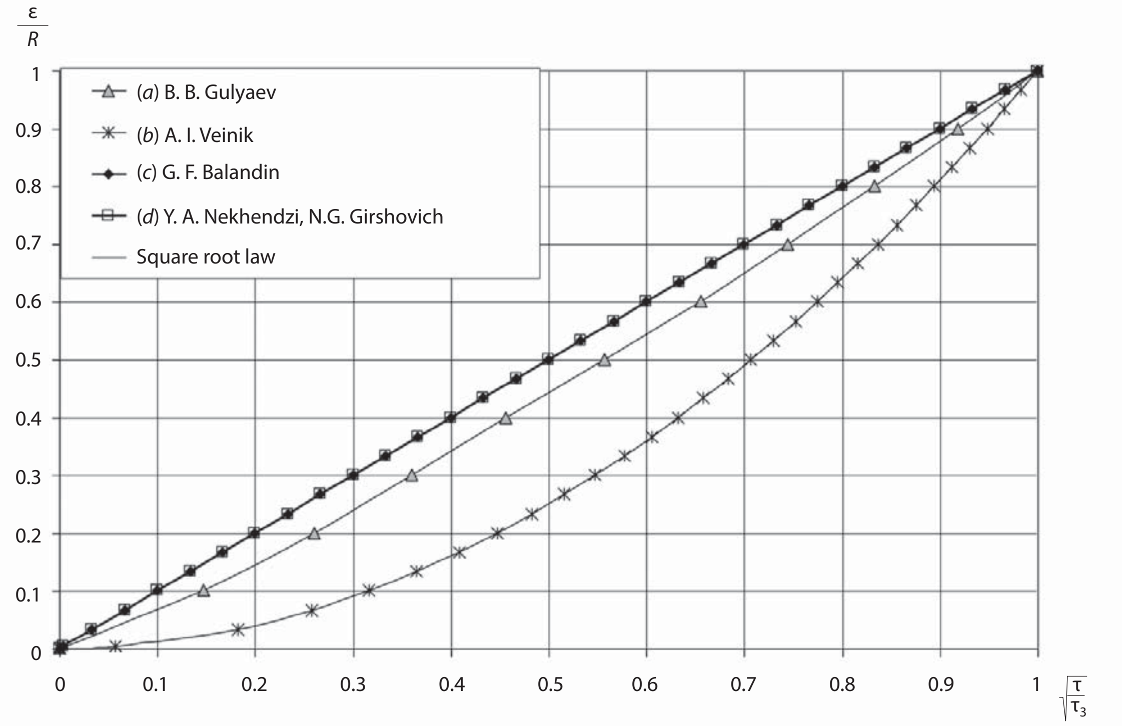

Fig. 1. The results of the calculation of the solidification boundary using theoretical and analytical approaches for castings of the "plate" type

thickness for castings of different configurations and the casting mold used.

The results are shown on the curves in dimensionless coordinates "relative thickness of the solidified skin (ε / R) – the square root of the relative time ($\sqrt{\tau / \tau_3}$)", where ε is the thickness of the solidified skin at the moment of time τ; R is the effective size of the casting, calculated as the ratio of casting volume V_0 to casting surface area F_0 ($R = V_0/F_0$); τ_3 is the total solidification time of the casting.

The specified coordinates characterize the formation of the solidified skin according to the square root law, with the process being compared to a straight line on the diagonal of the graph. The methodology used aims at comparing the studies in an easy-to-understand scale and approximating the available deviations of the actual solidification boundary from the square root boundary.

Results of calculations and analysis

The results of calculating the solidification boundary using theoretical and analytical approaches for castings of different configurations are shown in **Fig. 1–3**.

Summarizing the studies shown in Fig. 1, we can see that the theoretical curves of castings of the "plate" type are characterized by one kink, and the curves are located near the diagonal of the graph. Analyzing the results obtained by Yu. A. Nekhendzi and N. G. Girshovich, we must conclude that their curve corresponds well to the diagonal of the graph, and this applies to castings of all configurations. If we look at the curve of G. F. Balandin, we can see its compliance with the square root law. Particular attention should be paid to the curve of V. V. Gulyaev, which is located on the graph slightly below the diagonal. The latter can be explained by the fact that the rate of solidified skin growth at the beginning of the solidification process is lower in comparison with the rate of solidified skin growth determined by the square root law, while in the final period of solidification the rate increases.

According to A. I. Veinik, the theoretical curve lies well below the diagonal and has an inflection at the relative solidification time ($\sqrt{\tau / \tau_3}$)≈ 0.65.

In Fig. 2, we can observe the casting solidification curves calculated by A. I. Veinik and G. F. Balandin, which correspond to the cylinder configuration and are located below the diagonal of the graph. All curves are characterized by a certain increase in the movement of the solidification boundary, especially near the end of the process.

The theoretical curve of solidification of a cylindrical casting by V. V. Gulyaev up to ($\sqrt{\tau / \tau_3}$) < 0.6 is close to the diagonal, then deviates to the lower half of the graph, the last portions of the melt should also solidify at a higher rate.

Fig. 3 shows the results of calculating the solidification boundary using theoretical and analytical approaches for castings of the "ball" type. It can be seen that the theoretical curves' trajectory coincides very well among different researchers. At the same time, the curves have two kinks and are located below the diagonal of the graph. Also, we observe an interesting fact that during solidification, the solidification boundary in castings of the "ball" type at approximately 30-40% of the solidified melt volume moves at a higher rate than at the beginning of the solidification process.

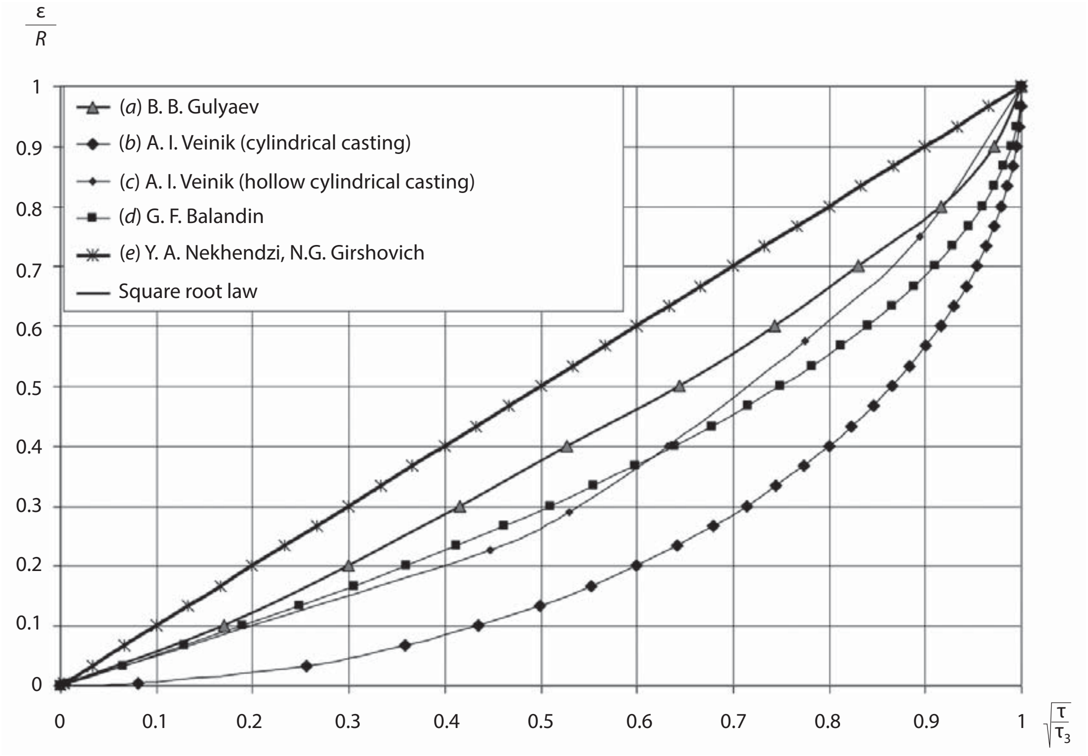

Fig. 2. The results of the calculation of the solidification boundary using theoretical and analytical approaches for castings of the "cylinder" type

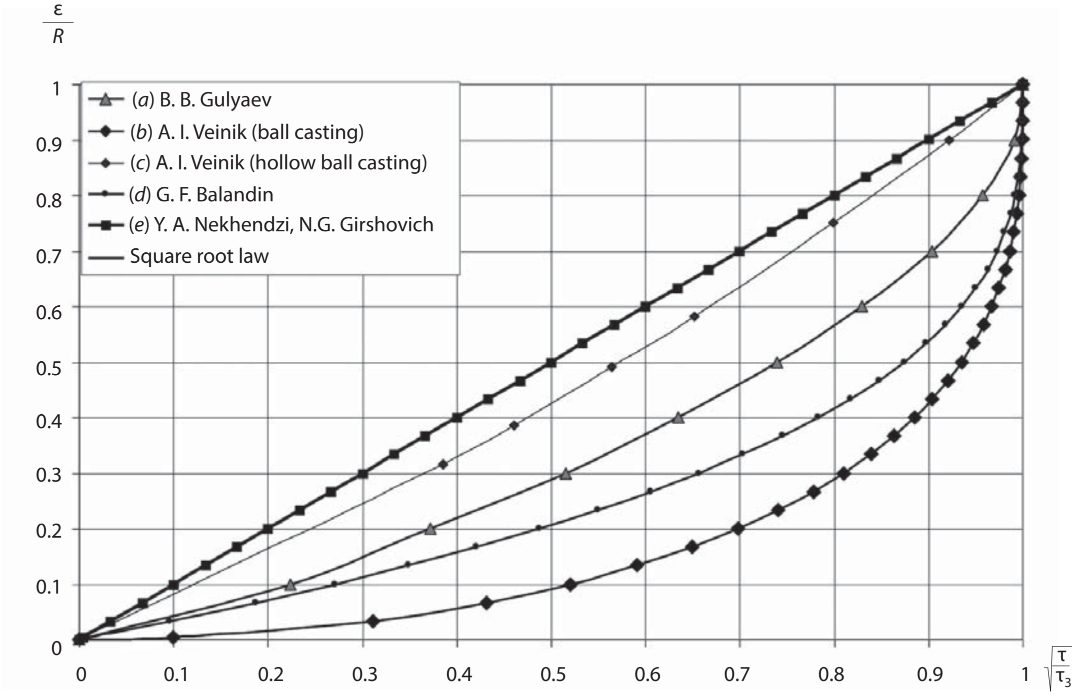

Fig. 3. The results of the calculation of the solidification boundary using theoretical and analytical approaches for castings of the "ball" type

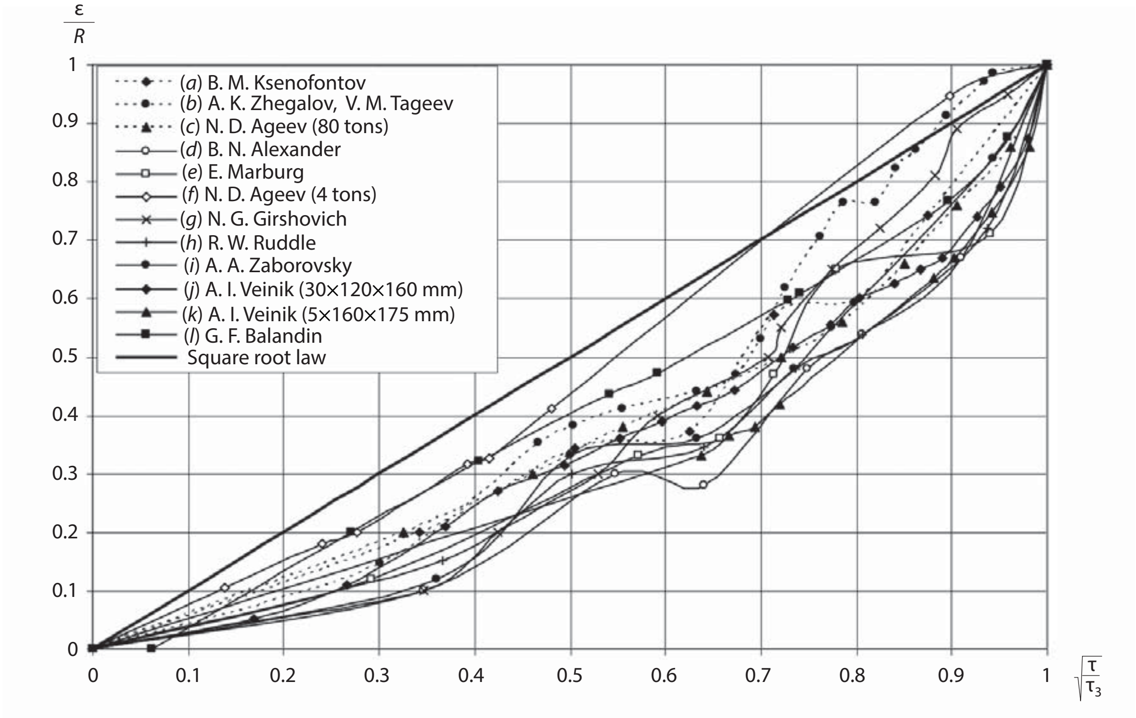

Fig. 4. Determination of the thickness of the solidified skin of castings during solidification in a metal mold

The results shown in **Fig. 4** characterize the determination of the thickness of the solidified skin of castings according to the methods of different researchers during solidification in a metal mold. The results were analyzed on different alloys using different methods of solidified skin thickness study, with castings having different weights. It should be noted the scatter of experimental data, and the trajectories of all curves are located below the diagonal of the graph.

The results shown in **Fig. 5** characterize the determination of the thickness of the solidified skin of castings according to the methods of different researchers, during solidification in a sand mold. Compared to the results in Fig. 4, we can

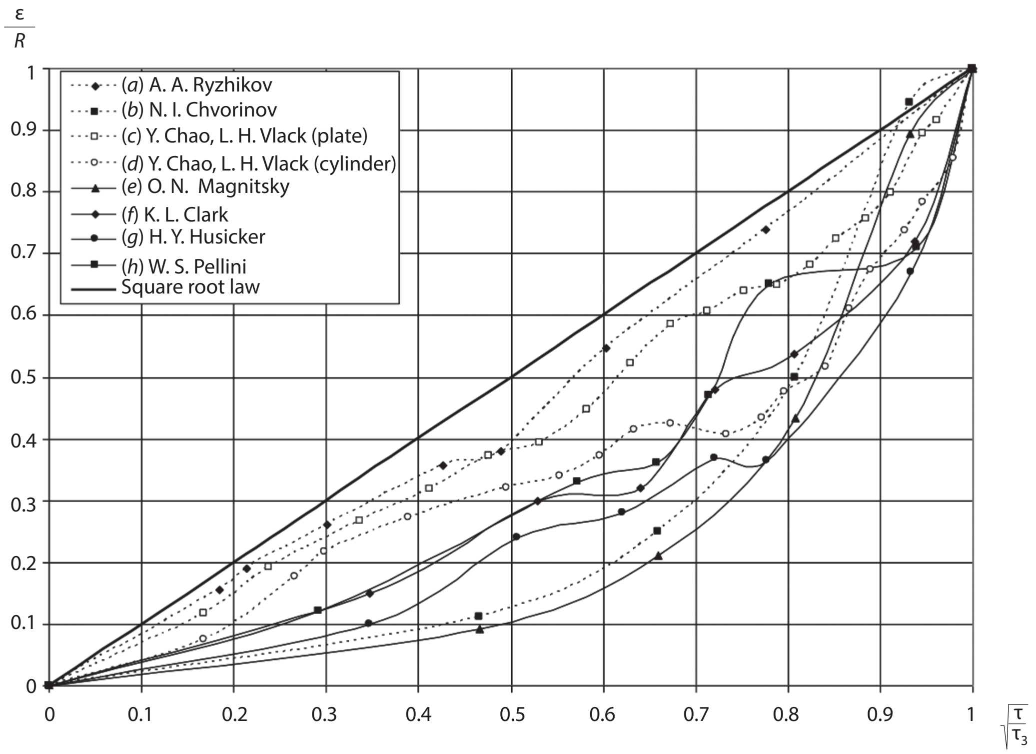

Fig. 5. Determination of the thickness of the solidified skin of castings during solidification in a sand mold

see a fairly large scatter in the data obtained. This can most likely be explained by the imperfection of the experimental methods used. It should be noted that the movement of the solidification boundary is located below the diagonal line of the graph.

Fig. 6 illustrates a comparison of the calculation of different solidification parameters of castings using numerical simulation and known empirical expressions. It can be seen that there is practically no scatter of points on the graph, and they correspond to the obtained curve lines. This can be explained by the fact that the thickness of the solidified skin itself was determined by calculation.

We can observe the kink in the curve obtained by the model of A. I. Veinik, which is typical for most other studies during the solidification process of castings in molds with very different heat removal (this applies to both metal molds and sand molds).

A comparison of some calculations shows that the methodology of Yu. P. Nekhendzi – N. G. Girshovich allows us to obtain a curve close to the theoretical curve of A. I. Veinik for cylinder-shaped castings. At the same time, the P. P. Berg and P. G. Novikov methods make it possible to obtain a curve close to the theoretical curve of A. I. Veinik for plate-shaped castings.

Empirical expressions for calculations proposed by V. M. Golod, A. A. Ryzhikov, N. I. Chvorinov, L. S. Konstantinov, and S. Schwartz are based on expressions for the square root law, so they allow to obtain results that are very close to the theoretical curve of the square root law.

During interpretation of the data shown in Fig. 1—5, a natural question arises: why do the practical results of the study of solidification processes in the given coordinates lie below the diagonal line (i.e., below the curve of the square root law)?

The following hypothetical mechanism can be proposed to explain it. At the initial stage of melt solidification in the mold, crystal growth dominates, mainly in the form of dendrites ($\varepsilon/R \approx$ 0.0-0.24). In the second stage, in addition to the continued growth of dendrites, new crystallization centers larger than the critical size nucleate in the liquid melt volume and begin to grow independently ($\varepsilon/R \approx$ 0.24-0.62). In the final stage, solidification of the melt occurs closer to the volumetric character of crystallization in the mold ($\varepsilon/R \approx$ 0.62-1.0). Due to the release of latent heat of melt crystallization at the initial stage of solidification in the mold, its total solidification time increases, which causes a deviation of the actual experimentally obtained values from the square root law. The second stage, by its thermophysical parameters, is the closest to the square root law, which can be observed quite clearly in the graphs. At the third stage, the process of solidified skin growth is avalanche-like, with a steeper slope. To confirm this hypothesis, additional experiments are required to study the processes of solidification of classical bodies of castings from different types of alloys.

The presented results of a critical review and generalization of investigations in the field of solidification of castings have an important applied value for determining rational technological parameters of the production of high-quality castings of different configurations in sand and metal molds.

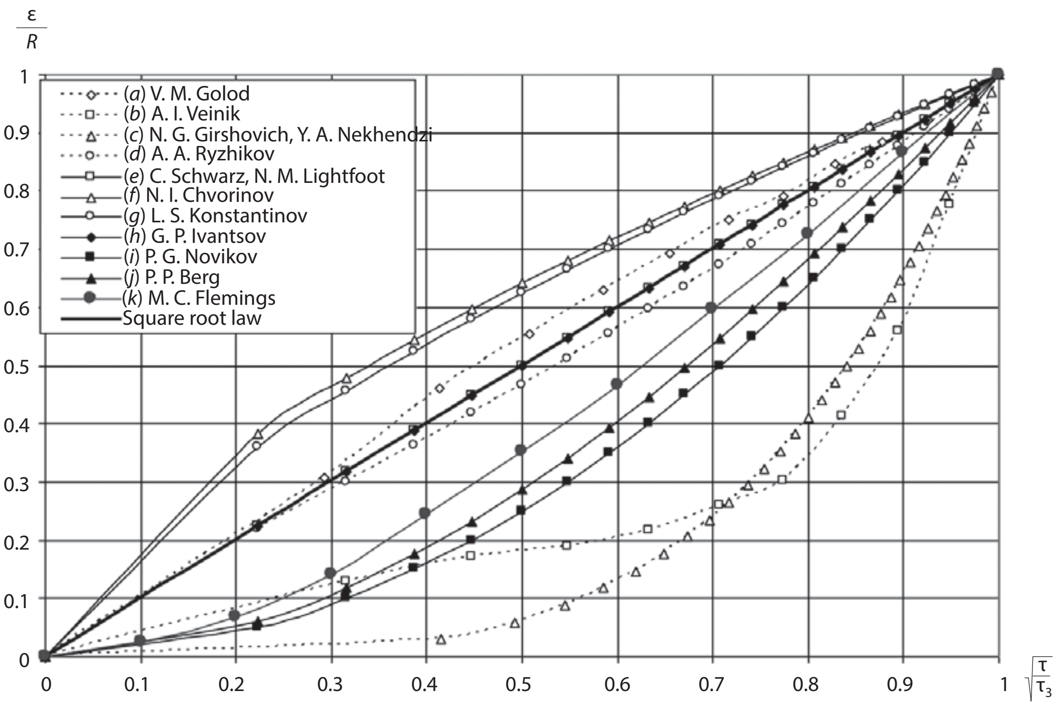

Fig. 6. **Comparison of calculated data for different solidification parameters of castings using numerical simulation and known empirical expressions**

In particular, specific economic effects from the implementation and reasonable choice of engineering methods of casting solidification parameters estimation can be achieved, first of all, by reducing the amount of defective castings through controlled purposeful influence on the thermal and thermostressed state of the "casting-mold" system and increasing the stability of casting quality in industrial processes, and by increasing the accuracy of casting production cycle duration determination, which will eventually increase the production capacity of foundry plants. Further research and development in this direction will contribute to the formation of practical recommendations for managing the processes of solidification of castings, including through the application of various methods of external influences on the processes of crystallization of metals and alloys.

Conclusion

A comprehensive analysis of curves in dimensionless coordinates showed that all numerical models and empirical expressions for analytical calculations of solidification parameters can be divided into three groups. The first group should include those models and empirical expressions, the calculation by which allows us to obtain results close to the calculated results according to the square root law. The second group includes numerical models and empirical expressions, the calculation by which allows us to get results that are higher than the calculated results under the square root law. And the third group includes models and empirical expressions, the calculation of which leads to results closest to the theoretical solidification curves of castings of the typical configuration (plate, cylinder, ball).

Based on the results of the analysis, a hypothetical mechanism of the process of casting solidification was proposed, which makes it possible to explain the stages of formation of their structure and the nature of the deviation of the experimentally obtained values of the solidification parameters of castings from the square root law.

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REFERENCES

1. Veinik A. I. Theory of Casting Solidification. Moscow: Mashgiz, 1960.
2. Balandin G. F. Fundamentals of the theory of formation of castings. Part 1. Moscow: Mashinostroienie, 1976.
3. Chvorinov N. Solidification of castings. Moscow: Izd. inostr. lit., 1955.
4. Gulyaev V. V. Solidification and heterogeneity of steel. Leningrad – Moscow: Metallurgizdat, 1950.
5. Veinik A. I. Thermal bases of the theory of casting. Moscow: Mashgiz, 1962.
6. Girshovich N. G., Nekhendzi Yu. A. Analytical solution of simple problems on solidification of castings of different configuration. *Liteinoe proizvodstvo*. 1956. No. 3. pp. 14-19; No. 4. pp. 13-17; No. 6. pp. 13-17; No. 12. pp. 13-18.
7. Stefanescu D. M. Science and Engineering of Casting Solidification, New York: Springer-Verlag, 2015. 559 p.

8. Dantzig J. A., Rappaz M. Solidification, Boca Raton, FL: CRS Press, 2009.
9. Fredriksson H., Akerlind U. Solidification and Crystallization Processing in Metals and Alloys. John Wiley & Sons, Ltd., 2012.
10. Flemings M. C. Solidification Processing, New York: McGraw-Hill, 1974.
11. Chvorinov N. Crystallization and heterogeneity of steels. NCSAV, Prague, Czech Republic, 1954. [in Czech].
12. Ryzhikov A. A. Technological fundamentals of foundry production. Moscow: Metallurgizdat, 1950.
13. Ruddle R. W. The Solidification of Castings. Inst. of Metals, Lnd. 1957. 116 p.
14. Ono A. Solidification: The Separation Theory and its Practical Applications, New York: Springer, 1987.
15. Temkin D.E. (ed.) Liquid metals and their solidification. Moscow: Metallurgizdat, 1962.
16. Golod V. M. Theory of foundry processes. Leningrad: Leningradskij politekhnicheskij institut, 1983.
17. Deev V. B., Prusov E. S., Shunqi M., Ri E. H., Bazlova T. A., Temlyantsev M. V., Smetanyuk S. V., Ponomareva S. V., Vdovin K. N. The influence of the melt cooling rate on shrinkage behaviour during solidification of aluminum alloys. *IOP Conf. Ser.: Mater. Sci. Eng.* 2019. Vol. 537. Iss. 2. 022080.
18. Cambell J. The New Metallurgy of Cast Metals, Second Edition, Butterworth Heinemann 2003.
19. Chvorinov N. Theorie der erstarrung von gussstücken. Giesserei. 1940, S. 177-186.
20. Prikhodko O. G., Deev V. B., Prusov E. S., Kutsenko A. I. Influence of Thermophysical Characteristics of Alloy and Mold Material on Casting Solidification Rate. *Steel in Translation*. 2020. Vol. 50. Iss. 5. pp. 296–302.
21. Souza E. N., Cheung N., Santos C. A., Garcia A. The variation of the metal/mold heat transfer coefficient along the cross section of cylindrical shaped castings. *Inverse Problems in Science and Engineering*. 2006. Vol. 14 (5). pp. 467–481.
22. Skočilasová B., Skočilas J. Effect of mold material on temperature distribution in alloy cast. *AIP Conference Proceedings*. 2016. 1768. 020017.
23. Pariona M. M., Mossi A. C. Numerical simulation of heat transfer during the solidification of pure iron in sand and mullite molds. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*. 2005. Vol. 27 (4). pp. 399-406.
24. Zhang A., Liang S., Guo Z., Xiong S. Determination of the interfacial heat transfer coefficient at the metal-sand mold interface in low pressure sand casting. *Experimental Thermal and Fluid Science*. 2017. Vol. 88. pp. 472-482.
25. Zhang L., Li L. Determination of heat transfer coefficients at metal/chill interface in the casting solidification process. *Heat and Mass Transfer*. 2013. Vol. 49. pp. 1071-1080.
26. Dargusch M.S., Hamasaiid A., Dour G. An inverse model to determine the heat transfer coefficient and its evolution with time during solidification of light alloys. *International Journal of Nonlinear Sciences and Numerical Simulation*. 2008. Vol. 9 (3). pp. 275-282.
27. Vandersluis E., Ravindran C. Influence of solidification rate on the microstructure, mechanical properties, and thermal conductivity of cast A319 Al alloy. *Journal of Materials Science*. 2019. Vol. 54. 4325–4339.
28. Abdallah Z., Aldoumani N. Casting Processes and Modelling of Metallic Materials. London, IntechOpen, 2021.
29. Luo A. A. Advanced Metal Casting. In: Caballero F.G., ed. Encyclopedia of Materials: Metals and Alloys, Vol. 3. Oxford: Elsevier, 2022. pp. 13–26.
30. Wang T., Wei J., Wang X., Yao M. Progress and application of microstructure simulation of alloy solidification, *Acta Metall. Sin*. 2018. Vol. 54. No. 2. pp. 193–203.
31. Guo J., Samonds M. Modeling of alloy casting solidification. *JOM* . 2011. Vol. 63. pp. 19–28.
32. Stefanescu D. M., Suarez R., Kim S. B. 90 years of thermal analysis as a control tool in the melting of cast iron. *China Foundry*. 2020. Vol. 17. pp. 69–84.
33. Santos R. G., Garcia A. Thermal behaviour during the inward solidification of cylinders and spheres and the correlation with structural effects. *International Journal Cast Metals Research*. 1998. Vol. 11. pp. 187-195.
34. Havlicek F., Elbel T. Geometrical modulus of a casting and its influence on solidification process. *Archives of Foundry Engineering,* 2011. Vol. 11 (4). pp. 170-176.
35. Auger J.-M. Local definition for surface-based geometrical modulus and use in automated casting cooling determination. *International Journal of Cast Metals Research*. 2017. Vol. 30 (6). pp. 322-336.