

Optimal distribution of capsules with active substance for the crack detection system in a turbine blade body

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This research is devoted to optimization of the detection system for damages of blades of a gas turbine engine in non-stationary conditions. The system is based on the approach on the method, when capsules with active substance are placed in a steel blade body, then a capsule is bursting due to difference of pressures during the process of crack propagation; as a result, ejected active substance is registered. The problem of this research is connected with optimal distribution of capsules, which contain active substance, in a turbine blade body. The research technique was developed for a modeling problem, when a steel turbine blade with constant cross section, having tensile crack and subjected to the action of extension centrifugal force, was observed. Development of the model with optimal distribution of capsules was based on assessment of pre-critical crack growth: it was required to distribute the capsules along the median line of blade cross section in such way, that it would be possible to detect the crack of pre-critical growth. Analysis of ultimate state of a blade with crack was carried out on the base of stress intensity coefficient, which allows to take into account the feature of crack location and to determine its critical length when its accelerated growth takes place. Variation of the pressure inside a capsule, required for its bursting depending on opening of crack sides. The suggested model takes into account location of the capsule with active substance relating to crack opening sides. Calculation of minimal number of capsules in blade body depending on pressure inside the capsule and rotating speed of blades was conducted on the example of the steel turbine blade R-5530 B. The obtained dependences allow us to find the optimal combination between the geometric and physical characteristics of the crack detection system and the minimal number of capsules. The application of the proposed approach to the optimal distribution of capsules along the median line of the blade cross section will increase the efficiency of use of the damage detection system in turbine blades in non-stationary conditions, thereby ensuring the safety of operation of gas turbine engines in aviation technology.

Key words: transverse separation crack, steel, turbine blade, stress state, damage detection system, cross section, gas turbine engines.

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Introduction

At present time, owing to rise of the operating temperatures and high rotation speed of rotors in gas turbine engines, the problem of cracks detection in turbine rotating blades seems more and more actual. Up-to-date rotating and nozzle blades of gas turbine engines are among the most responsible components of turbine equipment. Appearance of various internal or surface defects in blades can have effect on operation of turbines in working procedure. The problems of cracks detection in blades of gas turbine engines are rather actual during recent years, because they are connected with operating safety of aircraft engineering. Despite the fact that different ways of solving this problem are suggested, the problem of cracks detection in non-stationary operating conditions of turbine blades is not solved yet. There are many methods for cracks detection in turbine blades using online monitoring, vibration analysis, X-ray testing [1–3]. However, they are not valid for assessment of state of blades in stationary conditions. The main methods of non-destruction control of blades of aircraft engines are examined in details in the works [4, 5]. The technique and

technology of destruction diagnostics in aircraft turbine blades are considered in the researches [6, 7]. Early assessment of defects in the components of turbine equipment is also one of directions in revealing damages at present time [8]. Possibility of diagnostics of damages in turbine blades and the methods of non-destruction control are observed in the works [9, 10].

As soon as cracks can appear due to various kinds of deformation, the nature of cracks origination owing to creep and thermal fatigue on the example of turbine disks and blades via the method of finite element analysis [11, 12]. Assessment methods of stress state of disks, turbine blade and interlocks of turbine equipment are considered in the researches [13, 14]. As soon as turbine blades, especially rotating blades, are operating in the conditions of cyclic loading, the problem of fatigue destruction is very important. Prediction of fatigue parameters of turbine blades with damages are examined in the work [15]. The causes of cracks forming during fatigue loading, appearance of single cracks in blades during multiple loading, decrease of fatigue destruction resistance ability in the process of cyclic loading were investigated in the researches [16, 17].

Additionally, the important questions concern simulation of cracks propagation and assessment of dominating destruction mechanisms of turbine blades. The key problems of the crack formation theory are examined in the works of D. Nott, W. Broek, K. Hellan, N. Morozov [18–21]. Analysis of blade destruction mechanics based on cracking is considered in the research [22]. Peculiarities of crack growth in a nozzle blade are showed in the study [23]. The problems of mathematical destruction and mathematical simulation of cracks in turbine blades are displayed in the articles [24, 25]. Thermal loading of thermal blades, the problems of heat supply and optimization of heat removal are considered as one more cause having the effect on crack propagation process; corresponding investigation is presented in [26].

Thus, according to analysis of technical literature sources, the main methods of damages diagnostics using vibration analysis, X-ray images etc. are efficient only for analysis of turbine blades in non-operating conditions. There are a lot of works which are devoted to examination of causes of cracks formation, their growth and development models; at the same time, the problems of detection of blades damages for operating gas turbine engine are not studied practically. The existing methods and technologies are usually directed on cracks detection in stationary conditions. In this case, in the process of pre-critical growth, a crack is not always detected, while tearing-off of a blade part during operation of a gas turbine engine can lead to significant damages. Respectively, the problem of cracks detection in turbine blades appeared for non-stationary conditions. For this purpose, the researchers in their works [27–30] examined the features of cracks propagation and suggested the system for detection of damages in a turbine blade for operating engine conditions. According to these researches, the capsules with active substance, which displays ionizing properties at increased temperatures, are placed inside blade body. When crack sides in the area of a capsule are opening due to pressure gradient, tearing-off occurs and active substance is thrown in a flow-through part of turbine and is registered there. According

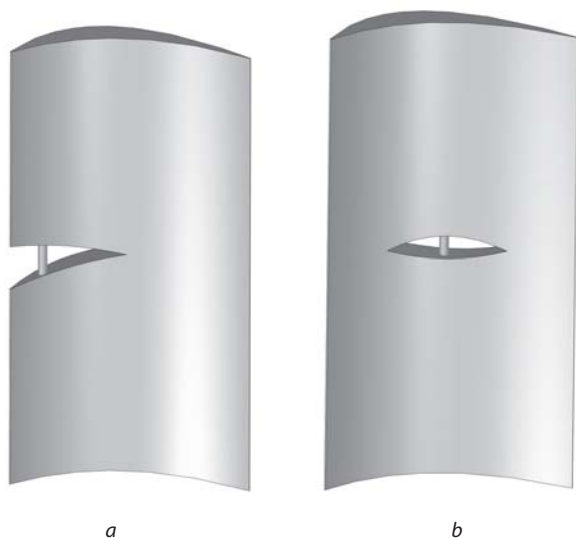


Fig. 1. Turbine blade scheme with a capsule during crack growth:
a – in the area of entrance edge; *b* – inside blade body

to the conducted experiments [27–30], the suggested approach has possibility of practical realization. However, the problem of optimal number of these capsules in a blade body is still not investigated, because insufficient number of capsules can lead to the fact that propagating crack can not meet the incorporated capsule, and tearing-off of a blade part can occur before registration of damage. At the same time, superfluous number of capsules can require additional material expenses, what is not substantiated.

Thus, the aim of this work is development of the approach for calculation of optimal capsules distribution in a blade body, in order to provide detection of cracks with pre-critical growth. It is important to mention, that the process of crack propagation is rather complicated, and it is not always possible to predict possible direction of crack propagation, depth of crack penetration and width of its opening. Additionally, the material of turbine blades can be heterogeneous, and inclusions, cavities and micro-cracks of different kind can cause damage origination. Various operating conditions of turbine blades, different geometrical configurations can also influence on the features of crack propagation. Owing to complication of accounting of the total diversity of the factors, it is suggested to solve a modeling task to calculate minimal number of capsules in a blade body for detection of cracks during loading process. Solving of a modeling task will be carried out on the base of several allowances and simplifications, which are listed below.

1. A turbine blade is solid, its cross section remains constant.
2. An examined turbine blade is under the effect of extension centrifugal forces. Operating turbine blades in actual conditions are subjected to action of centrifugal forces and bending distributed loads from gas flow. As soon as the values of bending forces, as a rule, is seriously smaller that extension centrifugal forces, we shall not take them into account.
3. Propagation of transversal tensile crack in a blade body is considered. This choice is stipulated by the fact, that such type of cracks is met more often in turbine blades and is the most dangerous, because it can lead to tearing-off of a blade part due to action of extension centrifugal forces. In this relation, longitudinal cracks are not so dangerous.
4. Tensile crack propagation occurs in the direction of a median line of a blade shape, what is stipulated by crack growth perpendicular to direction of maximal extension stresses. Crack surface points are shifting in the direction perpendicular to direction of crack surface.
5. A capsule which is placed in longitudinal direction of a blade body, is a cylinder thin-walled shell.
6. The 3rd strength theory is used for assessment of strength conditions of a capsule located near the opening area of crack sides. This choice is stipulated by the thesis, that destruction of a capsule occurs in the area of crack opening due to shell shearing, when tangential stresses achieve ultimate values.

Goal setting

Based on the assumed allowances, it is required to develop the model of capsules distribution along the median line of steel blade shape (crack opening near the capsule on

the Fig. 1 is displayed for better visualization, in real conditions motion of crack sides can be smaller than 1 mm). It should be taken into account in this case, that the required number N of capsules depends on the length l and width v of opening of crack sides, pressure p , which is created inside a capsule; it needs to consider the optimization task with the following restrictions:

$$N \rightarrow \min_{l, \delta, p \in D}, D: F(l, \delta, p) = 0.$$

Optimal number of capsules will be determined from the condition of detection of a crack with pre-critical growth

$$N_* = \min N = N|_{l=l_*}$$

where $2l_*$ – critical crack length.

At present day, the stress intensity coefficient K , which allows to take into account shape and dimensions of a body, location and length of a crack, features of loading, is one of the main parameters to characterize crack growth:

$$K = \sigma \sqrt{\pi l} Y,$$

where σ – extension stress, $2l$ – crack length, Y – K -calibration.

When studying crack growth in the area of entrance or exit edges (Fig. 1a), the authors used the Gross model [31] to calculate K -calibration of a crack on the body boundary under the effect of extension forces:

$$Y = 1.12 - 0.23\lambda + 10.55\lambda^2 - 21.72\lambda^3 + 30.39\lambda^4,$$

where $\lambda = l/L$, L – length of a median line of blade shape.

To assess crack growth inside blade body (Fig 1b), K -calibration is carried out according to the Ishida model [31]:

$$Y = \sqrt{\pi} [1 + 2.38\lambda^2 + 0.481\lambda^4].$$

To determine the critical crack length l_* , which corresponds to spontaneous crack propagation, the condition of start of quick crack growth is used: $K \geq K_{Ic}$, where K_{Ic} – toughness of material destruction. When the intensity coefficient of destruction toughness is achieved, accelerated crack growth occurred. The corresponding critical crack length is determined from the following condition:

$$\sigma = \sqrt{\pi l_*} Y(l_*) = K_{Ic}. \tag{1}$$

The equation (1) is non-linear and, respectively, it has no analytical solution. Thereby the numerical Newton-Raphson can be used for calculation of critical crack length.

Use of the relationship (1) allows to assess crack critical length on the base of data about destruction toughness; however, information about destruction toughness can be found in technical and reference literature for not many alloys (mainly for steel alloys). In these cases natural experiments for K_{Ic} determination are required.

According to the research [32], stress state in operating turbine blade with constant cross section, initiated by centrifugal extension forces, does not depend on cross section square and is determined by the following relationship:

$$\sigma = \frac{\rho \omega^2}{2} (R^2 - z^2),$$

where ρ – density of blade material; ω – angular rotation speed; z – distance between rotation axis and selected cross section; R – external radius of blade ring.

Research technique

Let us determine the number of critical lengths of cracks, which are located on the median line of the shape: $m = [L/l_*]$, where [...] is a “ceiling” function, rounding up the value of a non-integral number. If we call the number of capsules for the crack critical length as n , then the number of capsules along the median line of blade cross section will be determined by the following relationship:

$$N = [L/l_*](n - 1) + 1.$$

Let us emphasize the small element of the capsule cylinder shell (Fig. 2) near opening of crack sides (Fig. 3) and consider the equilibrium condition for a capsule shell element:

$$(2r\delta + \delta^2)\tau = 2rpv, \tag{2}$$

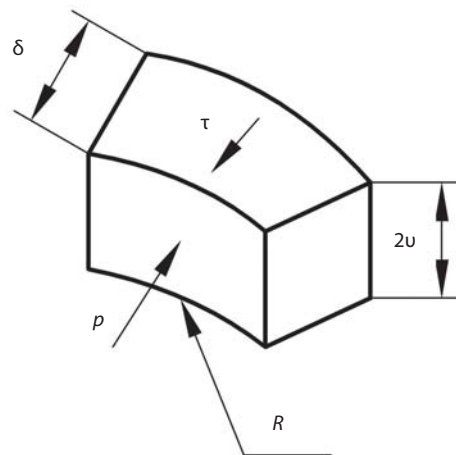


Fig. 2. The element of the capsule shell near crack opening

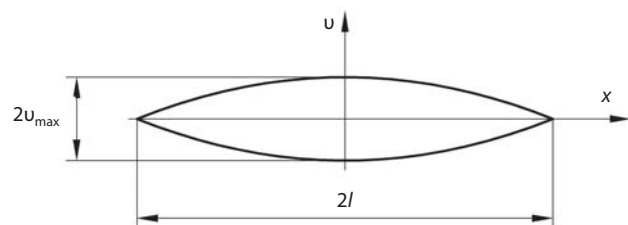


Fig. 3. Opening of crack sides

where r – internal radius of cylinder capsule; δ – capsule thickness; p – pressure inside a capsule; τ – tangential stresses.

Using the 3rd strength theory, we can see that the shear stress value in the case of capsule tearing-off can reach $\tau \geq \sigma_B / 2$, where σ_B is material tensile strength. Based of the relationship (2), the value of internal pressure in a capsule is determined by the following dependence:

$$p_* = p \geq \sigma_B \left(1 + \frac{\delta}{2r} \right) \frac{\delta}{2v}. \quad (3)$$

The required internal pressure p , which is required for capsule tearing-off in the case of crack growth, depends on radius and wall thickness of a capsule as well as on opening value of crack sides. Let us introduce the coordinate system Ovx , locating its beginning in the area of maximal crack opening (see Fig. 3).

Vertical motion of a crack side points at $|x| \leq l$ is calculated as follows (according to [31]):

$$v = \frac{2\sigma}{E} \sqrt{l^2 - x^2}, \quad (4)$$

where E - elasticity module of blade material.

Maximal crack opening is achieved at $x = 0: v_{\max} = 2\sigma l / E$. If the capsule is located in the area of maximal opening of a crack with critical length at $x = 0$, then the pressure, which is required for capsule tearing-off, is determined according to the dependencies (3), (4) from the following condition:

$$p_{\min} = \frac{\sigma_B E}{4\sigma} \left(1 + \frac{\delta}{2r} \right) \frac{\delta}{l_*}. \quad (5)$$

However, if the capsule is located at $x \neq 0$, then the value of crack opening will decrease with $|x|$ enlargement. Based on the obtained relationship (3), in the case of decrease of the crack opening area, the required pressure will increase in accordance to the calculated value (5), but it will be insufficient for capsule tearing-off.

Thus, we shall consider that capsules are homogeneously distributed along the median line of shape, with constant step Δl , which is determined by distance between capsules. If we suggest that this distance is equal to crack critical length $\Delta l = 2l_*$, then the case when capsules will locate in crack tops is possible and, respectively, pressure inside a crack will be insufficient for its tearing-off (Fig. 4a). taking into account that location of cracks can't be predicted beforehand, let us consider the case when distance between capsules is smaller than critical length. As soon as this distance Δl decreases, number of capsules for crack critical length increases. The closer is a capsule to the area of maximal crack opening, the smaller is internal pressure which is required for capsule tearing-off.

Let us consider variation of capsule location with the relative coordinate $|\bar{x}_c| = x_c / (2l_*)$, as most close to the area of crack maximal opening, depending on the number n of capsules, which are accounted on the crack critical length (Fig. 4).

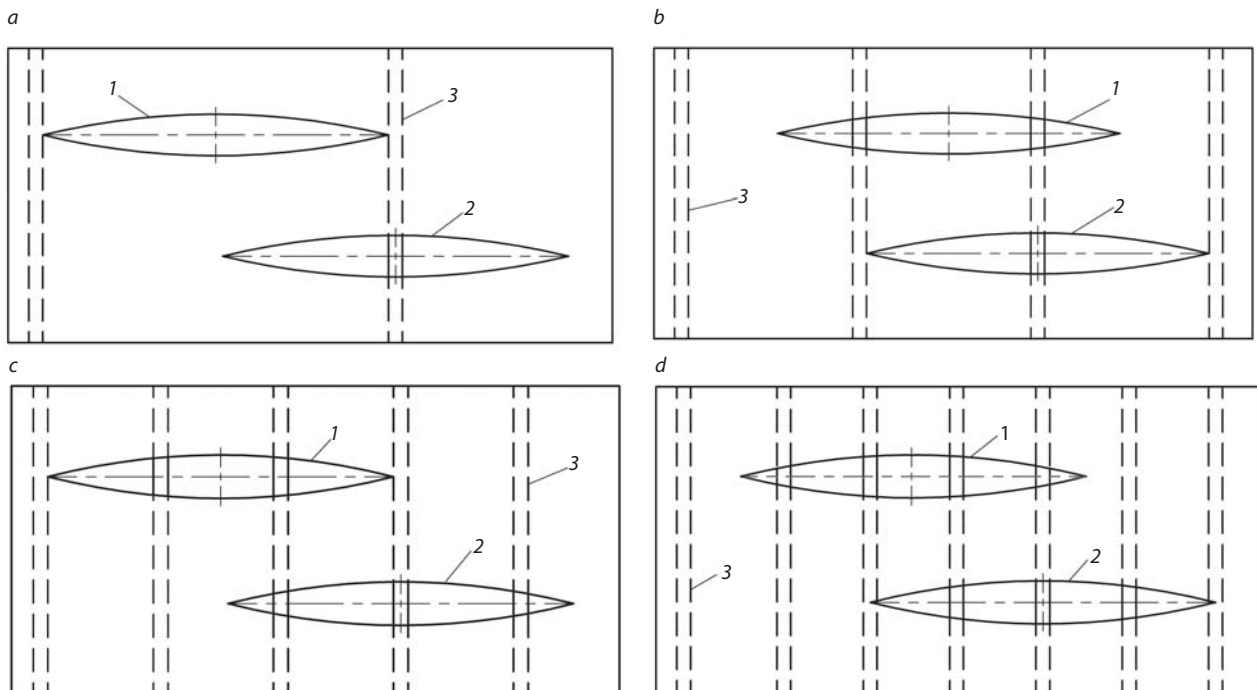


Fig. 4. Scheme of capsules distribution in the blade body with crack, depending on relative distance between capsules $\Delta \bar{l} = \Delta l / 2l_*$:
 a – $\Delta \bar{l} = 1$; b – $\Delta \bar{l} = 1/2$; c – $\Delta \bar{l} = 1/3$; d – $\Delta \bar{l} = 1/4$;
 1 – «unfavourable» location of capsules and crack; 2 – «favourable» location of capsules and crack; 3 – capsule

$$\left. \begin{aligned} \text{when } n = 2 : \Delta l = 2l_*, |\bar{x}_c| \in [0; 1/2] \\ \text{when } n = 3 : \Delta l = l_*, |\bar{x}_c| \in [0; 1/4] \\ \text{when } n = 4 : \Delta l = 2l_*/3, |\bar{x}_c| \in [0; 1/6] \\ \text{when } n = n_* : \Delta l = 2l_*/(n_* - 1), |\bar{x}_c| \in [0; 1/[2(n_* - 1)]] \end{aligned} \right\} \cdot (6)$$

Capsule location with the minimal relative coordinate $|\bar{x}_c|$ is considered as “unfavourable”, if it is located at maximal distance from the place of maximal crack opening. If the capsule is located in the area of maximal crack opening at $\bar{x}_c = 0$, then such state is assessed as “favourable” (see Fig. 4). Thus, “unfavourable” capsule location relating to the place of maximal crack opening is determined, according to the equations (6) by maximal value of $|\bar{x}_c|$ coordinate:

$$|\bar{x}_c| = 1/[2(n_* - 1)] \cdot (7)$$

Then crack opening near the «unfavourable» location of capsule will be described by the following relationship, in accordance with dependencies (4) and (7):

$$\bar{v} = \frac{v|_{x=\bar{x}_c}}{2l_*} = \frac{\sigma}{E} \sqrt{1 - \frac{1}{(n_* - 1)^2}} \cdot (8)$$

Using the expressions (3) and (8), we determine the required pressure inside a capsule for the case of «unfavourable» location of capsule relating to a crack:

$$p_* = p_{\min} \left[1 - \frac{1}{(n_* - 1)^2} \right]^{-1/2} \cdot (9)$$

Solving the equation (9) in relation to the number of capsules, which are accounted for the crack critical length, we shall obtain:

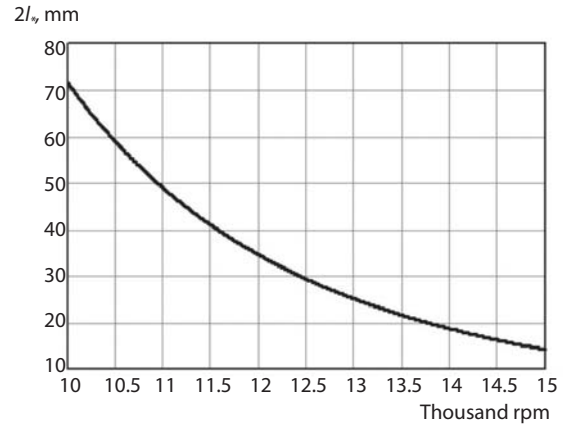


Fig. 5. Relationship between the crack critical length and blade rotation speed

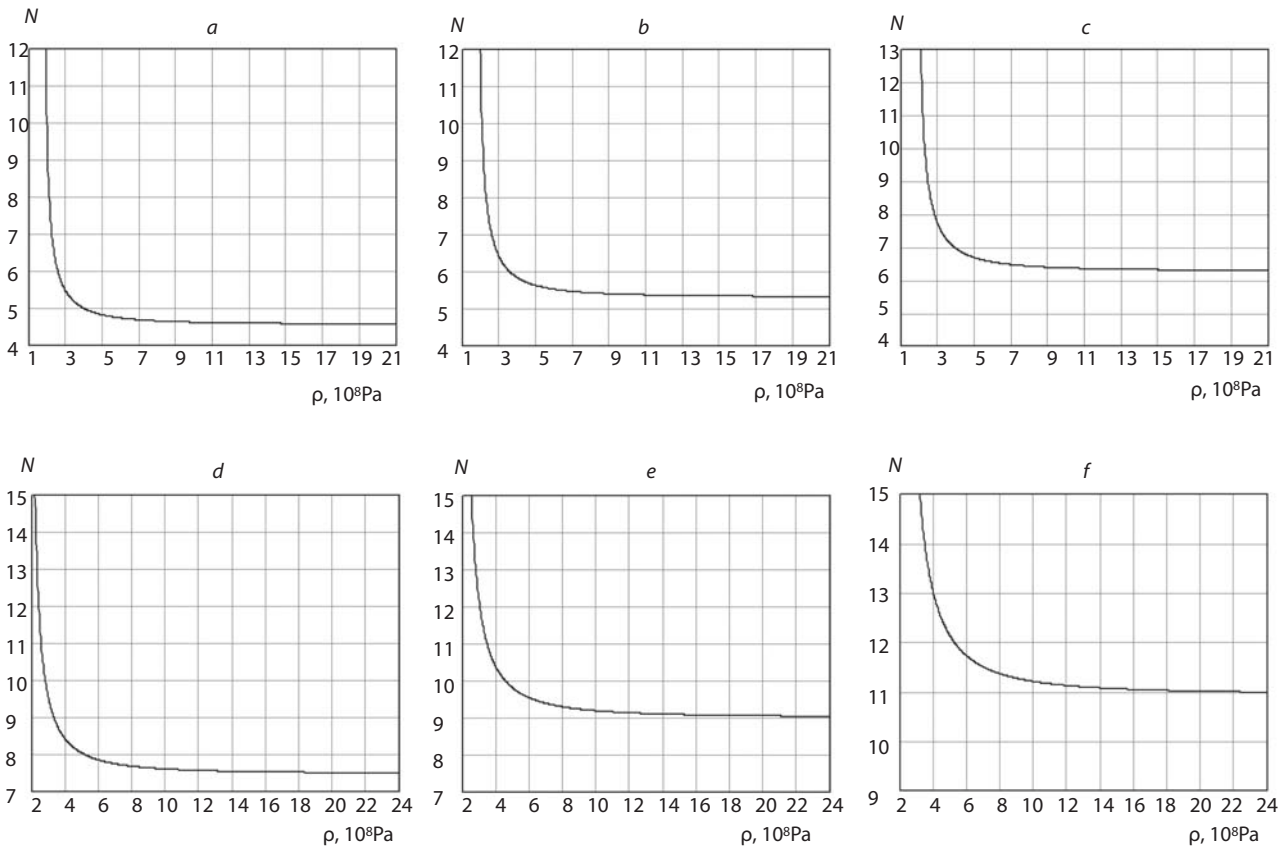


Fig. 6. Relationship between number of capsules and internal pressure inside capsules for different values of blade rotation: a – 10,000 rpm, b – 11,000 rpm, c – 12,000 rpm, d – 13,000 rpm, e – 14,000 rpm, f – 15,000 rpm

$$n_* = \left[1 - \frac{p_{\min}}{p_*} \right]^{-1/2} + 1.$$

Then optimal number N_* of capsules along the median line of blade shape is obtained from the following equation system (for the case $n = n_*$):

$$\left. \begin{aligned} N_* &= \left[\frac{L}{l_*} \left(1 - \frac{p_{\min}}{p_*} \right)^2 \right]^{-1/2} + 1 \\ p_{\min} &= \frac{\sigma_B E}{4\sigma} \left(1 + \frac{\delta}{2r} \right) \frac{\delta}{l_*} \\ \sigma &= \sqrt{\pi l_*} Y(l_*) = K_{1c} \end{aligned} \right\} \quad (10)$$

Solving of the non-linear equation system (10) allows to establish the dependence between the minimal number of capsules for diagnostics of damages and pressure value, which is required for capsule tearing-off during propagation of a crack pre-critical growth.

Calculation results

Let us consider the working turbine blade with shape R-5530 B [33] with the crack in the area of entrance edge. The steel SPN18K9M5T is selected as blade material, it has the following characteristics: $\rho = 7,700 \text{ kg/m}^3$, $E = 2.1 \cdot 10^5 \text{ MPa}$, $K_{1c} = 45 \text{ MN/m}^{3/2}$. The capsule shell is manufactured from aluminium alloy with tensile strength $\sigma_B = 100 \text{ MPa}$. Geometrical parameters of the capsule are such: $r = 2.5 \text{ mm}$, $\delta = 0.1 \text{ mm}$. Geometrical parameters of the blade and examined cross section are: $R = 0.3 \text{ m}$, $L = 0.1 \text{ m}$, $z = 0.25 \text{ m}$. Relationship between the crack critical length and rotation speed in the area of root cross section is presented for the preset parameters (Fig. 5).

The results of calculation of optimal number of capsules and pressure inside capsules for different values of blade rotation frequencies in a gas turbine engine, having the crack in the area of entrance edge, are presented on the Fig. 6.

Discussion of the results

According to the above-presented results (see Fig. 6), increase of pressure inside capsules in the system of turbine blade cracks detection leads to reducing of the minimal amount of capsules, which is required for diagnostics of cracks with pre-critical growth. The obtained relationships allow to observe convergence of number of capsules as a pressure function to any ultimate value. Taking into account that the number of capsules should be integral, it is observed that the minimal number of capsules varies from 5 to qq along the median line of cross section shape (within the examined range of turbine blade rotation frequencies 10,000–15,000 rpm). Rise of rotation speed leads to increase of the optimal number of capsules, what is caused by enlargement of load on a blade from centrifugal extension forces; decrease

of the critical length of a crack, which provides possibility of its accelerated growth, is also observed in this case.

Conclusion

The model of optimal capsules distribution for the system of damages diagnostics in turbine steel blades was built in this research for non-stationary conditions. The suggested approach of capsules distribution along the median line of blade cross section allows to provide detection of transversal cracks with pre-critical growth. The obtained system of equations can be applied in the case of development of transversal tensile cracks in the areas of entrance or exit edges, as well as inside a blade body. The presented optimization model also allows to determine optimal combination of internal pressure inside a capsule, its geometrical parameters and number. The described calculation results, which are based on the example of the operating turbine blade, make it possible to improve operation of the detecting system for damages of turbine blades for non-stationary conditions, thus increasing its efficiency in providing safe operation of gas turbine engines. It should be noted also that the problems of restrictions in capsules geometry and their influence on stress state of turbine blades with variable cross section, taking into account bending loads, were not examined in this research; they are the themes for additional investigation. CS

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