Calculation of the maximum functional clearance of a cylindrical joint between a steel shaft and a cast iron sprocket

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Cylindrical joints are widely used in mechanical engineering, where cast iron (sprockets of chain gears, gears, pulleys, etc.) act as the material for making holes, and steel is used for shafts. As a rule, these joints work in the conditions of dry and boundary friction and, in some cases, they have a key. The analysis showed that the wear of such joints occurred mainly due to the discrepancy between the accuracy standards, which, along with the strength standards, determine the durability and reliability of the joints. And if the strength standards are laid down by the designer, then the accuracy standards are usually assigned by analogy, based on the previous experience of using such constructions, with the use of software, when designer inserts standard deviations of shaft and hole as well as fit in electronic drawings. The main factors determining the margin of safety are such characteristics of the material as the maximal stresses on the cut, crumpling, bending, twisting, etc. To determine the margin of accuracy, it is necessary to set such parameters as the largest functional clearance or interference. Currently, there is no method for calculating the maximum functional clearance for joints with boundary or dry friction. The authors obtained dependence for determining the maximal functional clearance by the criterion of exceeding the allowable normal pressure on the contacting surfaces. An example of calculating the accuracy parameters for connecting a shaft made of steel with a malleable cast iron sprocket is considered, for which the maximal functional clearance is determined and fits are assigned – the first of the preferred series, and the second according to the criterion of reducing the cost of finishing processing of the components.
Key words: accuracy, tolerance, fit, fit tolerance, allowable normal pressure, largest functional clearance, smallest functional clearance.
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Introduction

Guaranteed reserve of workability can be provided via two methods: creation of wear-resistant surfaces and increase of processing accuracy for components [1]. Many methods to provide wear resistance of components were developed [1–4]. But increase of workability of any joints does not allow to elevate resource of machines or units in general, because other joints restrict this resource, and calculation of tolerances with account of reliability is required [5, 6]. Cylinder joints with keys, where shafts are fabricated from carbon and alloy steels (such grades as St3, St4, St5, 35, 40, 45, 50, 40Kh, 40KhNMA, 25KhGT etc.), are widely used in machine-building. They are characterized by high strength, ability to surface and volumetric strengthening, and cylinder billets from these steels have good workability in machine tools. Pulleys, sleeves, sprockets of chain gears etc. are connected with shafts, they are fabricated most often from cast iron (grey, malleable, high-strength), such grades as SCh18 – SCh36, KCh30 – KCh60, VCh40 etc. These joints were examined for wear resistance, service life and reliability [7]. Analysis displayed that wear occurred mainly owing to inconvenience to accuracy requirements. Calculation of new fits allowed to increase resource by 2–10 times [8, 9]. However, the first task is to determine clearances, within their range a joint operates well [10]. And if the minimal clearance is determined based on possible deviation of shape and surface location, thermal deformations, then calculation of the maximal clearance for each joint is completely individual [11, 12]. After determination of ultimate clearances, it is required to choose the fit and to substantiate the joint accuracy reserve. At present time, fit choosing is executed with use of software [13, 14]; this software uses the hole system (H) for the examined cylinder joint with key. Clearances and interferences in this system are forming via variation of the letter of shaft standard deviation and accuracy degree (**Fig. 1**), according to the acting GOST 12080-66. It is also allowed to use other deviations from standard values of the ISO Unified system of tolerances and fits, and designer decided by himself, which fit to be applied.

From one side, such approach decreases substantially operating workability at the designing stage. But from other side, it does not take into account possibility of creation of the preset accuracy reserve. It can be seen from the Fig. 1, that designer can form fits with clearance, fits with interference and transition fits. There is the actual question – which fit to be chosen, based on the criterion of joint service life? For example, it is required to provide quick disassembling and assembling of a joint in the field conditions, when failure of one of the assembling components takes place

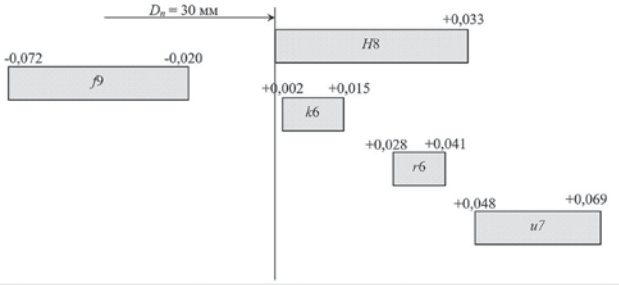

Fig. 1. Deviations and accuracy degrees of cylinder shaft ends, recommended by the GOST 12080-66

during agricultural operations; it means choosing the fit with clearance. Designer should choose, either $H8/f9$ fit, or $H8/h6$ fit, or any other joint (e.g. $H7/f6$ fit), which is included in the preferred values according to the ISO Unified system of tolerances and fits. All these fits will have initial clearances, consequently, they have different reserve of materials for wear. An here is another question: how to determine the ultimate clearance for evaluation of the value of materials for wear relating to this ultimate clearance?

The aim of this research is to find out mathematical relationship to determine maximal functional clearance of cylinder joint with key, without any lubrication in this joint.

Remedies and methods of research

The cylinder joint with key between steel 45 shaft and cast iron KCh30-6 sprocket, operating in the condition of dry and boundary friction, is the object of this research.

The clearance which is forming in this joint in the condition of dry and boundary friction is the subject of this research.

Theoretical investigation was carried out using the methods of analysis and synthesis of different scientific sources, devoted to the problems of determination of ultimate functional clearances in different joints of machines and assemblies. The basic issues of the theory of strength of materials, theory of accuracy and interchangeability were used during development of mathematical relationship in order to determine the ultimate functional clearance in the cylinder joint with key, without lubrication in this joint.

Results and discussion

Generalized parameters, which determine operation of a movable joint and influence on choice of clearance size, were systematized during analysis of operating requirements for such joints:

1) providing of relative slipping of conjugated surfaces with guaranteed reliability reserve (carrying ability for liquid lubrication, absence of jamming for dry and boundary friction etc.);

2) features of acting pressures and forces, as well as friction forces which hinder operating motion [14];

3) durability (wear resistance) of joint operation [5];

4) macro-geometry (deviations of dimensions, waviness) and micro-geometry (deviations of shape and location of surfaces) of joint components;

5) surface roughness of joint components [11];

6) temperature conditions of joint operation;

7) vibration resistance and damping properties of the joint;

8) fluid or gas consumption through the clearance, compressing properties;

9) partial factors which are typical for each concrete joint.

It can be seen, that there are many factors, and it is difficult to provide optimal combination of all of them. It is impossible to influence on construction during repair and maintenance of machines (parameters 1, 2, 6 and 7), so we can optimize the joint operating procedure via the effect on the parameters 3, 4 and 5, what will lead to essential rise of the joint durability.

Firstly the methods for calculation of optimal and ultimate clearances, which were based on hydrodynamic lubrication theory, were developed; its initial grounds were formulated by N. P. Petrov in 1833 on the base of O. Reynolds' researches. This theory was later developed by N. N. Zhukovskiy, S. A. Chaplygin, A. Sommerfeld, E. M. Gutyar, G. Fogelpol, A. Mitchell etc. Evaluation of clearance influence on joint parameters with use of hydrodynamic lubrication was investigated by M. V. Korovchinskiy, S. A. Chernavskiy, I. I. Pozdov et al. The technique for determination of ultimate functional clearances according to this theory is concluded in calculation via the following formula:

$$S_{F_{\min}^{\max}} = \frac{\omega \cdot k \cdot \mu \cdot d^2 \pm \sqrt{(\omega \cdot k \cdot \mu \cdot d^2)^2 - 16 \cdot p \cdot h_{\min}^2 \cdot \omega \cdot m \cdot \mu \cdot d^2}}{4 \cdot p \cdot h_{\min}}, \quad (1)$$

where ω — shaft rotation frequency, s^{-1}; d — joint diameter, m; p — specific pressure, Pa; $h_{\min}$ — minimal thickness of oil layer, m; μ — oil dynamic toughness, Pa·s; k and m — construction coefficients.

Determination of ultimate functional clearances for bearings with dry and semi-fluid friction is more complicated task.

A. I. Yakushev and V. N. Plutalov suggested the technique in their work [15], based on proving that the minimal functional clearance $S_{F\min}$ determines the minimal boundary of allowable clearance; in this case other operating parameters don't exceed the limits of allowable values. Calculation of $S_{F\min}$ means providing compensation of temperature and force deformations, technological errors, taking into account deviations of surfaces location for conjugated components:

$$S_{F\min} = (2h_l + \varphi l + \Delta_t)k, \quad (2)$$

where h_l — thickness of oil layer, m; φ — twist angle for hole and shaft axes, grad.; l — joint length, m; Δ_t — temperature deformations, m; k — slipping reliability coefficient for compensation of not accounted errors..

The maximal clearance is determined via the technique of Bauman Moscow State Technical University on the base of preset value of ultimate clearance, when the period of emergency wear takes place:

$$S_{\max} = S_{ult} - U, \quad (3)$$

where $S_{\max}$ — maximal clearance; S_{ult} — ultimate clearance; U — joint wear before appearance of the ultimate state. So

$U = (\gamma_d + \gamma_D)\, t$,

where γ_d, γ_D — wearing rate of shaft and hole surfaces respectively, μm/h; t — joint operating time (preset resource), h.

The maximal ultimate clearance for all kinds of joints can be determined individually via wearing dynamics [5, 6], but it is necessary for its building to collect statistic data on variation of hole and shaft dimensions, which are forming a joint. This determination can be carried out during operation, but it is connected with any negative features due to large time required for the experiment.

Above-mentioned relationships are valid for movable joints, when shaft rotates around a hole. But there are also stationary joints with clearance – joints with keys and splines, and it is important in this case to set the clearance when emergency wearing starts.

Cylinder joints with keys are very widely used in machine-building and serve mainly for transfer of small values of torques in different assemblies and drives. Contact of hole and shaft (with key) surfaces occurs during rotation of joint components around their axis (**Fig. 2**). The key impedes relative cranking in clearance, but micro-breakaway of roughness on the surface of components takes place owing to inequality of circumference length of a hole and shaft. Each rotating cycle is characterized by meeting practically the same points that contacted already with each other. Circulating loading is realized and, consequently, circulating uniform wear of hole and shaft surfaces is observed.

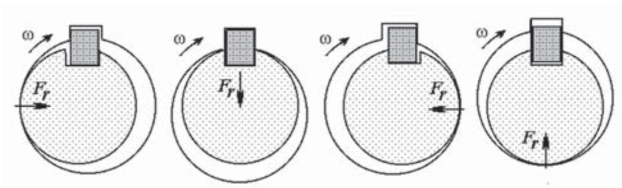

Fig. 2. Contacting of the surfaces during rotation of cylinder joint with key

The wearing process is influenced by the clearance value, as well as presence of abrasive material and lubricant in a friction area. The larger is clearance, the smaller is the contact square, and also we can observe increase of specific pressure, micro-breakaway rate, run-out, there is more strong contamination in the contact area, and wearing becomes more intensive as a result. Even if fit with interference is designed for joint in the construction, the clearance in this joint appears under the effect of radial force and console load, and the joint junction opens. In this case, uniform distribution of pressures along the joint is caused by radial force, while non-uniform distribution – by console load. Under combined effect of radial force and console load, the junction starts to open only in one part of the joint, but in any case it will be cyclic process. Shock-wave loading is initiated by appearing micro-breakaways, what leads to increase of wear and crumbling of surfaces in the "key – shaft groove – collar groove" joint in weak surface components (angles). A key imbalances in grooves and starts to acquire a round form. Jerks, flaps and slipping arise more and more, and complete crumbling ore key break-away can occur, and functional properties of a joint will be violated.

Reject of this or any other joint with clearance, without any lubricant, can be determined from the condition of non-exceeding the maximal normal stress σ_{max}. Actual contact square in this joint will decrease in the process of clearance increase, owing to diminishing of the contact area width. For linear contact, semi-axis of rectangular contact square (Fig. 2) will be equal to [16, 17]:

$$b = \sqrt{\frac{4R}{\pi l \sum \rho}\left(\frac{C_d}{E_d} + \frac{C_D}{E_D}\right)}, \quad (4)$$

where R – radial load, N; l – joint length, m; $\Sigma\rho$ – sum of curvature radii of contacting bodies, m; C_d and C_D – Lamé coefficient for shaft and collar; E_d and E_D – elasticity modules for shaft and collar materials, Pa.

$$\sum \rho = \frac{1}{r_d} - \frac{1}{r_D} = \frac{2}{d} - \frac{2}{D} = \frac{2D - 2d}{Dd} = \frac{2S}{d_n^2}, \quad (5)$$

where r_D and r_d – curvature radii of contacting bodies, m; d and D – shaft and hole diameters, m; S – clearance in the joint, m.

Maximal normal stress (**Fig. 3**) is equal to

$$\sigma_{max} = 2R/(\pi l b). \quad (6)$$

Extracting semi-axis of contact square b from the relationship (6) and substituting it in the left part of the expression (4), we shall obtain

$$\frac{2R}{\pi l \sigma_{max}} = \sqrt{\frac{4R}{\pi l \sum \rho}\left(\frac{C_d}{E_d} + \frac{C_D}{E_D}\right)},$$

And taking into account (5), we shall have

$$\frac{2R}{\pi l \sigma_{max}} = \sqrt{\frac{4R}{\pi l \frac{2S}{d_n^2}}\left(\frac{C_d}{E_d} + \frac{C_D}{E_D}\right)},$$

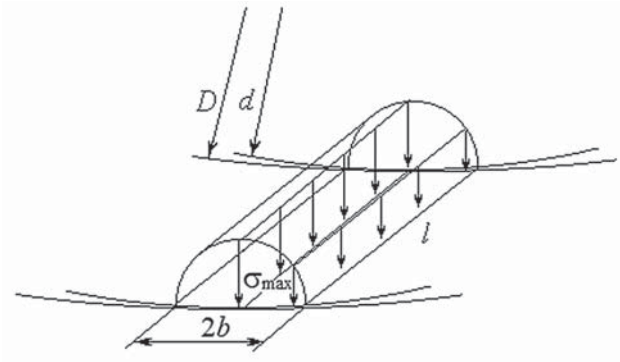

Fig. 3. Parameters of linear contact of the components

As a result of this solution, we shall obtain the formula for determination of the maximal functional clearance from the condition of non-exceeding the maximal normal stress:

$$S_{F\max} = \frac{\pi l d_n^2 \sigma_{max}^2}{2R}\left(\frac{C_d}{E_d} + \frac{C_D}{E_D}\right). \quad (7)$$

It is clear from the relationship (7), that exceeding the value of the maximal functional clearance by the actual clearance will lead to so large minimization of the contact square, that violation of the boundary of normal stress will occur. It will finalize in metal crumbling, break-away of its separate components and the third wearing stage (emergency wearing) will take place.

Table 1. **The data for calculation of construction parameters of a cylinder joint with clearance, having a key**

Parameters		Values
Nominal joint diameter, d_n, mm		30
Joint length, l, mm		45
Surface roughness	Sprocket holes, R_{aD}, µm	0.32
	Shaft holes, R_{ad}, µm	0.16
Load on the joint, R, kN		10.0
Allowable stress for crumbling, MPa	Shaft material – Steel 45	210
	Sprocket material – KCh30-6 cast iron	58
Lamé coefficients	of shaft C_d	0.98
	of sprocket hole C_D	2.38
Elasticity modules	of shaft E_d	$2 \cdot 10^{11}$
	of sprocket hole E_D	$1.72 \cdot 10^{11}$

Let us consider calculation of the maximal functional clearance on the example of a cylinder joint with a key (**Table 1**). This joint is mounted in the domestic potato combine harvester and take part in transit of motion and torque to operating mechanisms; it is located on the exit gear box shaft, which is connected with a chain gear sprocket. The shaft is manufactured from steel 45, while chain gear – from KCh30-6 cast iron. $H8/h9$ fit is used.

Table 1 data testify that the material of a collar (sprocket) ranks below significantly to the shat material (steel 45) in the value of allowable stress for crumbling, which was taken from [15], taking into account the conditions of alternative load.

Let us determine the value of the maximal functional clearance based on the relationship (7) and then we shall obtain $S_{F\max} = 401$ µm.

The minimal functional clearance for this joint is determined from the conditions of temperature deformations of components dimensions, which form the joint:

$$S_{F\min} = [\alpha_d \cdot (t_d - t) - \alpha_D \cdot (t_D - t)] \cdot d_n, \quad (8)$$

where α_d and α_D – coefficients of linear expansion of shat and collar material; t_d and t_D – operating temperatures of shat and collar; $t = 20$ °C – assembling temperature.

This clearance will be equal to 1.17 µm at the operating temperature 50 °C, because when heated, steel will expand more quick than cast iron, and this difference is required to be compensated. Actually, to provide quick assembling and disassembling of a joint, it is expedient to use the main shaft deviation $es = g = -7$ µm. It will be the minimal standard clearance in a joint, based on the condition of using the hole system. All results are presented in the **Table 2**.

It can be seen from the Table 2 that use of the preferable fit ∅ 30$H7/g6$ provides the coefficient of accuracy reserve larger the in use of the rational fit ∅ 30$H8/g7$, respectively, wearing reserve will be higher. But from the other side, use of the fit ∅ 30$H8/g7$ is more economically efficient, because in this case there is no need such processing accuracy as in use of the fit ∅ 30$H7/g6$. Finally, the choice will be done by designers and technologists of a machine-building enterprise. Comparison of the used and recommended fits in the values of initial clearances is presented on the **Fig. 4**.

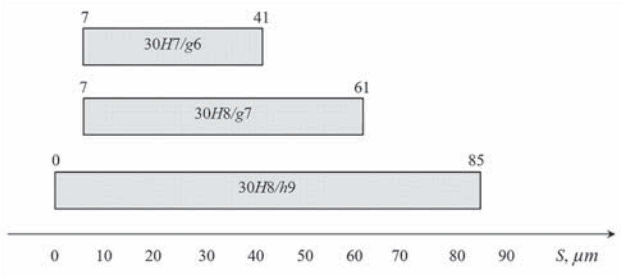

Fig. 4. Ranges of initial clearances in the used and recommended fits

Table 2. **The results of calculation of construction parameters and fits of a cylinder joint with clearance, having a key**

Parameters	Values
Maximal functional clearance, $S_{F\max}$, µm	401
Minimal functional clearance, $S_{F\min}$, µm	1.17
Main shaft deviation $es = g$, µm	7
When using the fit in the joint from the list of preferable ones	
Fit	30$H7/g6$
Maximal standard clearance, $S_{C\max}$, µm	41
Minimal standard clearance, $S_{C\min}$, µm	7
Fit allowance	34
Coefficient of accuracy reserve	11.6
When using the rational fit	
Fit	30H8/g7
Maximal standard clearance, $S_{C\max}$, µm	61
Minimal standard clearance, $S_{C\min}$, µm	7
Fit allowance	54
Coefficient of accuracy reserve	7.3

The obtained relationship (7) is valid for conjugated components made of ferrous metals (for the values of allowable crumbling stress) and can be used by engineers in designing of fits in cylinder joints with a key. These joints are operating in the conditions of boundary and dry friction, in order to determine the clearance value, which is typical for starting of emergency wear as well as for selection of a rational fit in the joint.

Conclusion

Theoretical relationship for determination of the maximal functional clearance via the criterion of exceeding allowable normal pressure on contacting surfaces was obtained on the base of geometric and strength analysis. Exit out of the boundaries of this pressure will lead to crumbling and deformation of the surface of more "weak" material of one of the component during contact, then the joint will transfer in the area of emergency wear. The example of calculation of accuracy parameters for joint connection between steel shaft and malleable cast iron sprocket; this joint was mounted on the exit end part of drive gear box of operating mechanisms of a potato combine harvester. It was established that crumbling will occur for hole material and the maximal functional clearance will be equal to $S_{F\max} = 401$ µm. The fits were determined for this joint: one preferential fit $\varnothing\, 30H7/g6$, and other fit $\varnothing\, 30H8/g7$, which will be characterized by lower manufacturing cost, because the components will be processed for more rough surface finish.

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