# THE MATHEMATICAL MODEL OF THE THICK STEEL SHEET FLATTENING ON THE TWELVE-ROLLER SHEET-STRAIGHTENING MACHINE. MESSAGE 1. CURVATURE OF SHEET 

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#### Abstract

The mathematical method for the determining of the optimal technological parameters of the cold straightening of the thick steel sheet on the twelve-rolled sheet-straightening machine is proposed. The calculations allow us to determine the type and the curvature of the neutral line of the steel sheet under the straightening depending on the rolls' radius, the pitch between the straightening machines' working rolls, the magnitude of the sheet reduction by the upper rollers, the sheet thickness, as well as the elastic modulus, the yield stress and the hardening modulus of the sheet metal. The results of the research can be used at the metallurgical plants.


## Introduction

After the hot rolling [1-12], the steel sheets are deforming during cooling due to the residual stresses and often have the surface defects in cold condition (for example, buckles, wavy edges, camber, crossbow, coil set and so on). Therefore, the steel sheets are flattened in the multi-roller straightening machines [1, 2, 13-28].

The process of sheet's straightening in the multi-roller flattening machines is mandatory (required) process for the technological processes of metallurgical production (fig. 1). The sheet flattening are widely used at Russian metallurgical plants (for example, in Vyksa, Chelyabinsk, Magnitogorsk, Izhora and so on) and at overseas metallurgical plants (in U.S., Germany, China, India and so on) [1-28].

The main task of the technology of the steel sheet straightening is to calculate the optimal reduction of a


Fig. 1. The multi-roller straightening machine by SMS Siemag
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sheet billet by the working rollers of straightening machines so that the sheet at the outlet from the machine has the minimum residual stress and curvature.

In the author's papers [25-28], the five-, eleven- and fifteen-roller straightening machine by Fagor Arrasate and the nine-roller straightening machines by SMS Siemag are investigated. This research paper is dedicated to the twelve-roller straightening machine (fig. 2).

We note that the shape of the neutral surface of the steel sheet during bending, the calculation of spring coefficients, residual stresses and critical pressures for various sheet defects, and the types of sheet metal forming are considered in the author's papers [20-22].

## Mathematical model of sheet flattening

Let $H_{i}(i=1, \ldots, 12)$ be the sheet's reductions by the working rollers. Without limiting the generality, further we assume that the lower six working rollers are fixed ( $\left.H_{1}=H_{3}=H_{5}=H_{7}=H_{9}=H_{11}=0 \mathrm{~mm}\right)$ and the upper six working rollers have the independent vertical movement.

Let $P_{t}$ and $P_{c}$ be the hardening modules of steel in tension and compression, $E$ and $\sigma_{y}$ be the young's modulus

of sheet's movement

Fig. 2. The sheet flattening in the twelve-roller straightening machine


Fig. 3. The kinematics scheme of sheet's flattening between working rollers
and yield strength, $R$ be the radius of the working rollers, $\varphi_{i}$ be the angles of the contact points of rollers with the steel sheet $(i=1 \ldots 12)$, $t$ be the step of the lower working rollers, $R_{0}=R+h / 2, \varepsilon_{i}=1 / \rho_{i}$ and $\rho_{i}$ be the sheet's curvature and the radii of sheet's curvature at the contact points of rollers with the steel sheet, $h$ be the sheet's thickness (fig. 3).

The mathematical equation for the calculations of the sheet's curvature at the points of the sheet's contact with the working rollers have the form

First and second rollers:
$z_{2}=\left(\frac{t}{2}-R_{0} \sin \varphi_{1}+R_{0} \cos \varphi_{2}\right) \cos \varphi_{1}+$
$+\left[H_{2}-R_{0}\left(1-\cos \varphi_{1}\right)-R_{0}\left(1-\cos \varphi_{2}\right)\right] \sin \varphi_{1}$,
$y_{2}=-\left(\frac{t}{2}-R_{0} \sin \varphi_{1}+R_{0} \cos \varphi_{2}\right) \sin \varphi_{1}+$
$+\left[H_{2}-R_{0}\left(1-\cos \varphi_{1}\right)-R_{0}\left(1-\cos \varphi_{2}\right)\right] \cos \varphi_{1}$,
$a_{1}=\frac{3 y_{2}+\operatorname{tg}\left(\varphi_{1}+\varphi_{2}\right) z_{2}}{z_{2}{ }^{2}}, \quad b_{1}=\frac{2 y_{2}+\operatorname{tg}\left(\varphi_{1}+\varphi_{2}\right) z_{2}}{z_{2}{ }^{3}}$,
$\varepsilon_{1,2}=2 a_{1}, \quad \rho_{1,2}=\frac{1}{2 a_{1}}$,
$\varepsilon_{2,1}=\frac{2 a_{1}-6 b_{1} z_{2}}{\left[1+\left(2 a_{1} z_{2}-3 b_{1} z_{2}^{2}\right)^{2}\right]^{\frac{3}{2}}}, \quad \rho_{2,1}=\frac{1}{\varepsilon_{2,1}} ;$
Second and third rollers:
$z_{3}=\left(\frac{t}{2}-R_{0} \sin \varphi_{2}-R_{0} \cos \varphi_{3}\right) \cos \varphi_{2}+$
$+\left[H_{2}-R_{0}\left(1-\cos \varphi_{2}\right)-R_{0}\left(1-\cos \varphi_{3}\right)\right] \sin \varphi_{2}$,
$y_{3}=-\left(\frac{t}{2}-R_{0} \sin \varphi_{2}-R_{0} \cos \varphi_{3}\right) \sin \varphi_{2}+$
$+\left[H_{2}-R_{0}\left(1-\cos \varphi_{2}\right)-R_{0}\left(1-\cos \varphi_{3}\right)\right] \cos \varphi_{2}$,
$a_{2}=\frac{3 y_{3}+\operatorname{tg}\left(\varphi_{2}-\varphi_{3}\right) z_{3}}{z_{3}^{2}}, \quad b_{2}=\frac{2 y_{3}+\operatorname{tg}\left(\varphi_{2}-\varphi_{3}\right) z_{3}}{z_{3}^{3}}$,
$\varepsilon_{2}=\varepsilon_{2,3}=2 a_{2}, \quad \rho_{2}=\rho_{2,3}=\frac{1}{2 a_{2}}$,
$\varepsilon_{3,2}=\frac{2 a_{2}-6 b_{2} z_{3}}{\left[1+\left(2 a_{2} z_{3}-3 b_{2} z_{3}^{2}\right)^{2}\right]^{\frac{3}{2}}}, \quad \rho_{3,2}=\frac{1}{\varepsilon_{3,2}} ;$
( $2 j-1$ )-th and $2 j$-th rollers $(j=2,3,4,5,6)$ :
$z_{2 j}=\left(\frac{t}{2}+R_{0} \sin \varphi_{2 j-1}-R_{0} \cos \varphi_{2 j}\right) \cos \varphi_{2 j-1}-$
$-\left[H_{2 j}-R_{0}\left(1-\cos \varphi_{2 j-1}\right)-R_{0}\left(1-\cos \varphi_{2 j}\right)\right] \sin \varphi_{2 j-1}$,
$y_{2 j}=\left(\frac{t}{2}+R_{0} \sin \varphi_{2 j-1}-R_{0} \cos \varphi_{2 j}\right) \sin \varphi_{2 j-1}+$
$+\left[H_{2 j}-R_{0}\left(1-\cos \varphi_{2 j-1}\right)-R_{0}\left(1-\cos \varphi_{2 j}\right)\right] \cos \varphi_{2 j-1}$,
$a_{2 j-1}=\frac{3 y_{2 j}-\operatorname{tg}\left(\varphi_{2 j-1}+\varphi_{2 j}\right) z_{2 j}}{z_{2 j}{ }^{2}}$,
$b_{2 j-1}=\frac{2 y_{2 j}-\operatorname{tg}\left(\varphi_{2 j-1}+\varphi_{2 j}\right) z_{2 j}}{z_{2 j}{ }^{3}}$,
$\varepsilon_{2 j-1}=\varepsilon_{2 j-1,2 j}=2 a_{2 j-1}, \quad \rho_{2 j-1}=\rho_{2 j-1,2 j}=\frac{1}{2 a_{2 j-1}}$,
$\varepsilon_{2 j, 2 j-1}=\frac{2 a_{2 j-1}-6 b_{2 j-1} z_{2 j}}{\left[1+\left(2 a_{2 j-1} z_{2 j}-3 b_{2 j-1} z_{2 j}^{2}\right)^{2}\right]^{\frac{3}{2}}}$,
$\rho_{2 j, 2 j-1}=\frac{1}{\varepsilon_{2 j, 2 j-1}} ;$
$2 j$-th and $(2 j+1)$-th rollers $(j=2,3,4,5)$ :
$z_{2 j+1}=\left(\frac{t}{2}+R_{0} \sin \varphi_{2 j}-R_{0} \cos \varphi_{2 j+1}\right) \cos \varphi_{2 j}-$
$-\left[H_{2 j}-R_{0}\left(1-\cos \varphi_{2 j}\right)-R_{0}\left(1-\cos \varphi_{2 j+1}\right)\right] \sin \varphi_{2 j}$,
$y_{2 j+1}=\left(\frac{t}{2}+R_{0} \sin \varphi_{2 j}-R_{0} \cos \varphi_{2 j+1}\right) \sin \varphi_{2 j}+$
$+\left[H_{2 j}-R_{0}\left(1-\cos \varphi_{2 j}\right)-R_{0}\left(1-\cos \varphi_{2 j+1}\right)\right] \cos \varphi_{2 j}$,
$a_{2 j}=\frac{3 y_{2 j+1}-\operatorname{tg}\left(\varphi_{2 j}+\varphi_{2 j+1}\right) z_{2 j+1}}{z_{2 j+1}^{2}}$,
$b_{2 j}=\frac{2 y_{2 j+1}-\operatorname{tg}\left(\varphi_{2 j}+\varphi_{2 j+1}\right) z_{2 j+1}}{z_{2 j+1}^{3}}$,
$\varepsilon_{2 j}=\varepsilon_{2 j, 2 j+1}=2 a_{2 j}, \quad \rho_{2 j}=\rho_{2 j, 2 j+1}=\frac{1}{2 a_{2 j}}$,


Fig. 4. The deflection of sheet

$$
\begin{aligned}
\varepsilon_{2 j+1,2 j} & =\frac{2 a_{2 j}-6 b_{2 j} z_{2 j+1}}{\left[1+\left(2 a_{2 j} z_{2 j+1}-3 b_{2 j} z_{2 j+1}^{2}\right)^{2}\right]^{\frac{3}{2}}}, \\
\rho_{2 j+1,2 j} & =\frac{1}{\varepsilon_{2 j+1,2 j}}
\end{aligned}
$$

The boundary conditions have the form

$$
\begin{aligned}
& \rho_{1,2}=\rho_{1} ; \quad \rho_{j, j-1}=-\rho_{j, j+1}, \quad j=2, \ldots, 11 ; \\
& \rho_{12}=\rho_{12,11}=\beta\left(\rho_{11,12}\right) \rho_{11,12},
\end{aligned}
$$

where the sheet's springback coefficient is equal to [2022, 25-28]

$$
\beta(\rho)=\frac{1}{\left(1-\frac{P_{t}+P_{c}}{2 E}\right)\left(1-2 \frac{\rho \sigma_{y}}{h E}\right)^{2}\left(1+\frac{\rho \sigma_{y}}{h E}\right)}
$$

## Numerical results

The numerical calculations for the deflection and curvature of the steel sheet in the twelve-roller straightening machine at $E=2 \cdot 10^{11} \mathrm{~Pa}, R=125 \mathrm{~mm}, \sigma_{y}=500 \cdot 10^{6} \mathrm{~Pa}$, $h=10 \mathrm{~mm}, t=270 \mathrm{~mm}, \rho_{1}=-1 \mathrm{~m}, H_{2}=12 \mathrm{~mm}$, $H_{4}=6 \mathrm{~mm}, H_{6}=3 \mathrm{~mm}, H_{8}=1.5 \mathrm{~mm}, H_{10}=0.75 \mathrm{~mm}$ and $H_{12}=0.375 \mathrm{~mm}$ are shown in fig. 4 and fig. 5.

The numerical calculations for the curvature of the steel sheet at $E=2 \cdot 10^{11} \mathrm{~Pa}, R=125 \mathrm{~mm}, \sigma_{y}=500 \cdot 10^{6} \mathrm{~Pa}$, $h=10 \mathrm{~mm}, t=270 \mathrm{~mm}, \rho_{1}=\infty \mathrm{m}, H_{2}=3 \mathrm{~mm}, H_{4}=6 \mathrm{~mm}$, $H_{6}=12 \mathrm{~mm}, H_{8}=6 \mathrm{~mm}, H_{10}=3 \mathrm{~mm}$ and $H_{12}=1.5 \mathrm{~mm}$ are shown in fig. 6.

## Conclusions

The analytical method for the determining of the curvature of the thick steel sheet under the cold straightening on the twelve-rolled sheet-straightening machine is suggested.

The results of the research can be widely used at metallurgical plants in the production of thick steel sheet, and the manufacture of the steel large- and medium-diameter tubes for the gas-oil main pipelines.

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Fig. 5. The curvature of sheet (the first case)


Fig. 6. The curvature of sheet (the second case)
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# THE MATHEMATICAL MODEL OF THE THICK STEEL SHEET FLATTENING ON THE TWELVE-ROLLER SHEET-STRAIGHTENING MACHINE. MESSAGE 2. FORCES AND MOMENTS 

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#### Abstract

The mathematical method for the determining of the technological parameters of the cold straightening of the thick steel sheet on the twelve-rolled sheet-straightening machine is proposed. The calculations allow us to determine the support reaction of working rollers, the residual stresses in the wall of the steel sheet, the proportion of plastic deformation on the sheet thickness and the relative deformation of the longitudinal surface fibers of the sheet under the straightening depending on the rollers' radius, the pitch between the straightening machines' working rollers, the magnitude of the sheet reduction by the upper rollers, the sheet thickness, as well as the elastic modulus, the yield stress and the hardening modulus of the sheet metal. The results of the research can be used at the metallurgical plants.


## Introduction

The rolling mills and the multi-roller straightening machines (fig. 1) are widely used for the steel sheet production in Russian and overseas metallurgical industry [1-24].

In the first part of the paper the technique of determining the shape and curvature $\varepsilon$ (radius of curvature $\rho$ ) of the steel sheet at flattening on the twelve-roller straightening machine was suggested.

In the second part of the paper we consider in detail the method of determining the bending moments and residual stresses, as well as the support reactions of the working rollers in the twelve-roller straightening machine.

We note that the estimation of the force parameters for the different methods of bending and forming in metallurgy are investigated in papers [10-21].

## The bending moments at the steel sheet flattening

Let $H_{i}(i=1, \ldots, 12)$ be the sheet's reductions by the working rollers. Without limiting the generality, further
we assume that the lower six working rollers are fixed $\left(H_{1}=H_{3}=H_{5}=H_{7}=H_{9}=H_{11}=0 \mathrm{~mm}\right)$ and the upper six working rollers have the independent vertical movement.

Let $P_{t}$ and $P_{c}$ be the hardening modules of steel in tension and compression, $E$ and $\sigma_{y}$ be the young's modulus and yield strength, $R$ be the radius of the working rollers, $\varphi_{i}$ be the angles of the contact points of rollers with the steel sheet $(i=1, \ldots, 12), t$ be the step of the lower working rollers, $R_{0}=R+h / 2, \varepsilon_{i}=1 / \rho_{i}$ and $\rho_{i}$ be the sheet's curvature and the radii of sheet's curvature at the contact points of rollers with the steel sheet, $h$ be the sheet's thickness.

Under the elastic bending $\left(\rho \geq \rho_{y}=h E /\left(2 \sigma_{y}\right)\right)$, the springback coefficient of steel sheet is equal to $\beta(\rho)=\infty$.

Under the plastic bending ( $\rho<\rho_{y}=h E /\left(2 \sigma_{y}\right)$ ), the springback coefficient of steel sheet is equal to

$$
\beta(\rho)=\frac{1}{\left(1-\frac{P_{t}+P_{c}}{2 E}\right)\left(1-2 \frac{\rho \sigma_{y}}{h E}\right)^{2}\left(1+\frac{\rho \sigma_{y}}{h E}\right)} .
$$

Under the plastic bending of steel sheet, the bending moment is equal to [10-12]

