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CALCULATION OF THE REQUIRED SEMIAUTOGENOUS MILL POWER BASED ON THE BOND WORK INDEXES

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Optimization of ore pretreatment and selection of the appropriate equipment is one of the main areas of activity of RIVS Science and Production Association.

For sizing ore pretreatment equipment, RIVS uses an integration of methods, including conventional commercial and semicommercial testing, and increasingly broadening laboratory testing of ore strength properties. Scale-up of the laboratory test data for the full-sized equipment is based upon mathematical modeling.

This article describes the method offered by Barratt for sizing circuit power requirements for autogenous and semiautogenous milling. The method is widely used by RIVS in extended calculations at early design stages.

The article is composed of sections successively informing on the existing techniques of sizing SAG mill power, Bond work indexes and their applicability to calculating autogenous/semiautogenous mills. Finally, the authors give a comparative analysis of the results obtained using the described method and in the joint commercial tests conducted by RIVS and SIBIR-Polymetals, Ural Mining and Metallurgical Company.

Decision-making on selection of SAG mills increasingly uses the methods of sizing the milling power based on laboratory tests on grindability of minerals. This article considers one of such methods allowing determination of mill characteristics based on the Bond work indexes of crushing, rod milling and ball milling. The authors exemplify the described method application and compare the obtained estimates with the factual field test data.

The article presents mathematical framework of a method used by RIVS to forecast energy input of various-type drum mills (ball mills, autogenous and semi-autogenous grinding mills). The basics of the described method were developed by S. Morrel, an Australian expert in mineral material milling, and have not been published previously in Russia. The RIVS experts have compared the new method and a few commonly used techniques and matched the forecasting values against the factual capacities of the on-site mills. As a consequence, it is concluded that the method described in this article yields the most accurate outcome due to the detailed accounting of ball charge motion.

The Introduction gives a description of the proposed approximation of ball charge position in a rotating drum and its dependence on the mill velocity. In what follows, the mathematical apparatus used for the forecasting calculation of capacities of the drum shell and cone is presented. The mathematical procedure is based on the calculation of the overall potential and kinetic energy of the ball charge during its motion inside the rotating mill. The concluding section of the article offers empirical formulas to calculate idling and aggregate capacity of the mill main drive.

Key word: *semiautogenous milling, modeling, strength properties, grindability, Bond work index.*

The ore pretreatment flow charts in use at most high-productive processing plants designed in the last decade are based on semiautogenous (SAG) milling and involve as a rule primary wet autogenous mill (WAM) and a series of ball mills.

WAM is customary chosen either based on examination of analogous equipment specifications, or from the evidence of very expensive commercial and semicommercial testing that requires large sampling, much time and high investment [1]. The current practice of exploratory design of ore pretreatment process increasingly uses laboratory testing of ore strength, which allows simple and inexpensive but reliable ranging of crushing and milling machines [2]. Three methods of lab testing are best suitable for sizing SAG mills and accurate determination of parameters of the mill drives:

- Barratt—Doll method based on the Bond work indexes [3];
- JK Drop Weight/SMC developed by JKMRC [4];
- Starkey’s method SAGDesign/SPI [5].

Each of the listed methods includes a series of laboratory disruptive tests with special-purpose equipment and a set of empirical equations for estimation of mill power sufficient for reaching required production level.

RIVS Science and Production Association has its own ore pretreatment laboratory equipped for full-fledged implementation of the first two methods in the list above.

All of the said methods aim to find specific energy for the required SAG comminution of test ore material.

Specific energy consumption (SEC) measured in kWh/t is a ratio of the mill drive power to the milling circuit output, without regard to circulating load. At the constant size of the cycle feed and discharge, SEC is a prime characteristic of material grindability in a certain ore pretreatment machine. For one kind material, SEC depends on the type of an ore pretreatment machine, which is connected with essential fluctuations of power transmission in active zone of various machines.

The product of SEC and the preset output yields the required grinding power, which conditions the choice of size and drive of a mill.

This article describes the Barratt–Doll method advantageous for being based on the Bond work indexes that are determinable in any well-fitted ore pretreatment laboratory (it is worth saying that JK Drop Weight and SAGDesign testing equipment is less common, especially, in the former USSR countries). One more benefit of the Barratt–Doll method is simple calculation and processing of lab test data, which eliminates the need for a dedicated program support.

Bond work indexes

In 1952 Bond published his famous article describing the procedure of ore estimation that is still the primary tool of design and optimization of crushing and grinding circuits [6]. There was no heretofore an adequate practical method of sizing ore pretreatment equipment based on its power.

According to Bond's theory, the energy consumed by crushing and grinding of mixture of ore particles is given by:

$$E = 10 \cdot \mathrm{WI} \cdot (1/\sqrt{P_{80}} - 1/\sqrt{F_{80}}), \qquad (1)$$

where P_{80} and F_{80} are the sizes of 80% ground (product) and initial (feed) material, respectively, µm; WI is the work index of material, kWh/t; E is the required specific energy, kWh/t.

The energy–size relation given above was derived by Bond empirically, after analysis of a lots and lots of commercial and laboratory test data on operation of rod and ball mills that he accumulated while working in Allis Chalmers, the world's top supplier of milling equipment those years. The highest concern of Allis Chalmers about reliable sizing of mills it supplied spurred the related research.

The wide-spread occurrence of Bond's theory is owing to Bond's method of ore strength assessment, that allows finding work indexes under laboratory conditions [4]. Theoretically, the work index equals the amount of energy (kWh/t) consumed to comminute 1 t of infinite size material down to 80% content of –100 µm size grade. The mechanism and nature of mineral comminution greatly depends on the size of the mineral particles; therefore, a work index is considered the function of size and is determined for definite size ranges using laboratory facilities fitting the mode of fracture. For instance, for large particles (comminution from 100 down to 10 mm), the crushing work index SWI is applicable; for medium size particles (from 100 mm to 2100 µm) — rod milling work index RWI; for finer particles — ball milling work index BWI (**Table**). These indexes are readily obtained using the special-purpose laboratory equipment (**Fig. 1**).

Standard size ranges and the related Bond work indexes

Feed size F_{80}, µm	Product size P_{80}, µm	Work index
100000	10000	crushing CWI
10000	2100	rod milling RWI
2100	100	ball milling BWI

Finding the total specific energy consumption in the wet autogenous mill — central discharge ball mill circuit (WAM — CDBM)

In the framework of the discussed method, total SEC in WAM — CDBM circuit is found based on total SEC in a conventional circuit including medium and fine crushing, and rod and ball milling. Upon comminution of the initial plant's feed (coarse crushing discharge) up to ball milling I discharge, the conventional circuit is divided into three size ranges in accordance with Table. Each size range has a specific Bond work index conditioned by the Bond theory.

Extensive discussion has been focused for many years on the energy hypotheses by Rittinger and Kirpichev–Kirck [7] for different researchers arrived at results which confirmed either that theory or the other. Publication of the "third" theory by Bond escalated the interest of practitioners in the energy aspects of ore pretreatment.

As of today, many publications are concerned with analysis of energy–size relations. The comprehensive survey and analysis has been performed to assess applicability of various methods to reliable estimation of the work indexes involved in these relations. The main inference is that the offered energy laws are only valid within the limited ranges of variables under specific conditions, although they ensure quite acceptable degree of extrapolation and interpolation of estimates obtained for the known equipment employed in the standard operating conditions [8].

For adaptation of Bond's law to the particular operating conditions, Rolland introduced 8 correction factors in detail described in literature in Russia [7] and often named as efficiency factors EF abroad. Currently, computation seldom uses these correction factors; as for the described method of total SEC consumption in the conventional circuit, only two correction factors are applicable — $EF4_{\mathrm{CDBM}}$ for initial size of ball mill feed and $EF5$ for final milling size; these factors are only introduced if exceed 1. For RWI and BWI in kWh/t, the formulas below are valid:

$$EF4_{\mathrm{CDBM}} = \left[1 + \frac{0.907 \cdot \mathrm{BWI} - 7}{F_{80}/P_{80}} \left[\frac{F_{80}}{4000\sqrt{14.33/\mathrm{RWI}}} - 1\right]\right]; \qquad (2)$$

$$EF5 = (P_{80} + 10.3)/1.145 P_{80}. \qquad (3)$$

It is worth noticing that calculation of $EF4_{\mathrm{CDBM}}$ requires RWI and BWI, and F_{80} in (2) is understood as the initial size of

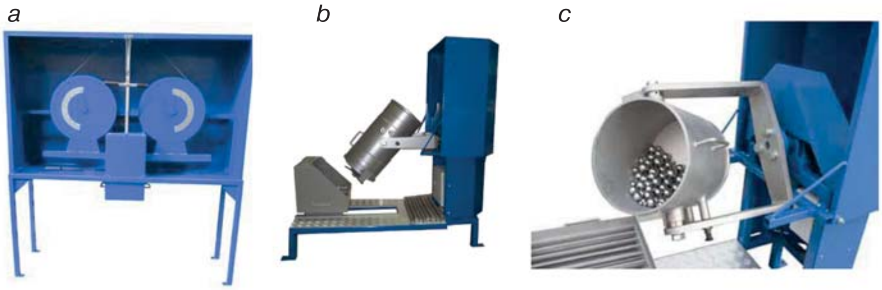

Fig. 1. Laboratory equipment used by RIVS for determining the Bond work indexes:
(*a*) CWI; (*b*) RWI; (*c*) BWI

feed for the rod mill, i.e. 10000 μm in accordance with Table. The value of P_{80} is accepted based on the required grain-size characteristics of the ball mill discharge.

Considering the Bond energy law and the correction factors by Rolland, the total specific energy E_{req} required for comminution of mineral from the initial size F_{80} (coarse crushing discharge) to the final size P_{80} (ball milling discharge) in the conventional circuit is found as follows:

$$E_{req} = 10\left[\mathrm{CWI}\left(\frac{1}{\sqrt{10000}} - \frac{1}{\sqrt{F_{80}}}\right) + \right.$$
$$\left. + \left[\mathrm{RWI}\left(\frac{1}{\sqrt{2100}} - \frac{1}{\sqrt{10000}}\right) + \mathrm{BWI}\left(\frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{2100}}\right)EF5\right]EF4_{CDBM}\right]. \quad (4)$$

The resultant Ereq governs the maximum efficiency of ore pretreatment equipment and fits the circuit of medium and fine crushing followed with rod and ball milling.

Barratt performed the comparative analysis of a wide range of ore pretreatment circuits and found that the SAG-based circuit requires energy 5–15% lower as against the conventional circuit discussed [9]. Accordingly, the total SEC for the same size milling with WAM — CDBM circuit is given by:

$$E_{total} = (1.05 \div 1.55) \cdot E_{req}. \quad (5)$$

Distribution of energy between WAM and CDBM mills

In 1989, within the framework of the conference on SAG milling, Barratt presented another formula for SEC required for WAM comminution of minerals from F_{80} to the size T_{80} [9]:

$$E_{WAM} = 10\left[\mathrm{CWI}\left(\frac{1}{\sqrt{P_C}} - \frac{1}{\sqrt{F_{80}}}\right) + \mathrm{RWI}\left(\frac{1}{\sqrt{P_R}} - \frac{1}{\sqrt{P_C}}\right)\times\right.$$
$$\left. \times EF4_{GDRM} + \mathrm{BWI}\left(\frac{1}{\sqrt{110}} - \frac{1}{\sqrt{P_R}}\right)EF4_{CDBM}\,EF5\right]\times \quad (6)$$
$$\times 1.25 - 10\mathrm{BWI}\left(\frac{1}{\sqrt{110}} - \frac{1}{\sqrt{T_{80}}}\right)EF4_{CDBM}EF5.$$

where T_{80} is the size of 80% content of ball mill feed in WAM — CDBM circuit (typically 1000–3000 μm); GDRM means grate discharge rod mill.

Barratt did not specified the ranges for the sizes P_C and P_R, but it is roughly possible to use the values from Table, i.e. P_C = 10000 μm and P_R = 2100 μm. The same values are applicable to calculating the efficiency factors $EF4_{GDRM}$ and $EF4_{CDBM}$. For example:

$$EF4_{GDRM} = \left[1 + \frac{0.907 \cdot \mathrm{RWI} - 7}{P_C/P_R}\left(\frac{P_C}{16000 \cdot \sqrt{14.33/\mathrm{RWI}}} - 1\right)\right]. \quad (7)$$

For $EF5$ the value of P_{80} is accepted based on the final size of the ball mill discharge in WAM — CDBM circuit.

In case that WAM — CDBM circuit additionally includes re-grinding of a critical size grade, SEC in re-grinding is considered the portion of E_{WAM}. Usually, it is very low (to 5%) and is neglected in calculation of WAM capacity.

Specific energy consumption for ball milling from T_{80} to P_{80} is calculated as the difference of the total energy consumption in WAM — CDBM circuit and E_{WAM}:

$$E_{CDBM} = E_{total} + E_{WAM}. \quad (8)$$

The advantage of this approach is accounting for increased yield of fine sizes in the SAG mill discharge as compared with the discharge of conventional crushers or rod mills operating at the same value of T_{80}. The product with the increased yield of fine sizes goes to sump of a ball mill and, bypassing the mill, is put out from the ball milling circuit with the discharge of cyclone–classifiers. As a result, the ball milling-required SEC is somewhat lower than is predicted in the direct application of the Bond law. In foreign literature, this phenomenon is named as a “phantom cyclone”.

Estimation of drive power in WAM — CDBM circuit

With the known E_{WAM} and E_{CDBM} (kWh/t), the required drive power N (kW) is found from multiplication process including the output Q (t/h). The drive power of a mill is customary understood as the power of the ring gear of the drum. The main drive transmits rotation to the ring gear by means of reducing gear or elastic coupling. The latter bring some mechanical energy loss, and the power of the main drive is therefore somewhat higher than the power of the ring gear.

Conventionally, the main drive of WAM is able to develop 90% of the installed power, and the efficiency of the reducing gear is 98.5%; accordingly, for the required installed power of the WAM main drive, we have:

$$N_{WAM} = \frac{E_{WAM} \cdot Q}{0.9 \cdot 0.985}. \quad (9)$$

A similar relation is valid for the ball mill with the only difference of a little bit higher efficiency of the drive (94%):

$$N_{CDBM} = \frac{E_{CDBM} \cdot Q}{0.94 \cdot 0.985}. \quad (10)$$

Given N_{WAM} and N_{CDBM} are known, relevant mills are chosen based on the comparative analysis of the developed power of various size mills. The description of the calculation method for developed powers is beyond the scope of this article and can be found in literature [4, 8].

Exemplification of the discussed method

In 2013 in the framework of technological regulations development for Rubtsovsk processing plant, Ore Pretreatment Laboratory, RIVS, accomplished strength testing of Stepnoe deposit ore, including determination of the Bond work indexes: CWI = 7.82; RWI = 22.95; BWI = 14.34 kWh/t.

Later on, Sibir-Polymetals and RIVS conducted joint commercial testing of Stepnoe deposit ore at Rubtsovsk processing plant (**Fig. 2**). The commercial tests yielded the following results:

At the overall plant output of 77 t/h, SAG milling achieved comminution from F_{80} = 102528 μm to T_{80} = 157 μm (55.5%

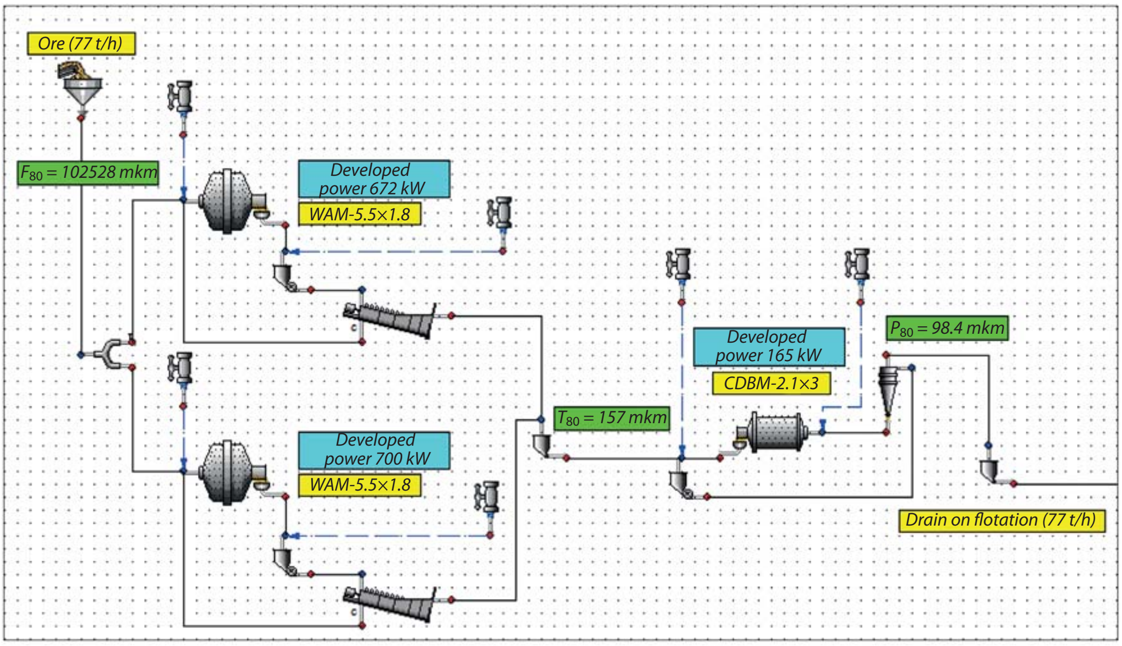

Fig. 2. Commercial testing equipment layout

content of size –74 μm); the recorded total developed power of two WAM-5.5×1.8 was 1372 kW;

At the ball milling stage (CDBM = 2100×3000), the joint mill classifiers discharge with the size T_{80} = 157 μm was reduced to the size P_{80} = 98.4 μm (66.5% content of size –74 μm); in conformity with the accepted operating mode, the developed power of milling was 165 kW;

The calculation of the required power in the WAM — CDBM circuit by the proposed method is reported below.

1. Specific energy consumption for the conventional circuit comminution from the size F_{80} = 102528 μm to the size P_{80} = 98.4 μm is found from Eq. (4) with taking into account the correction factors (2) and (3), for which $EF4_{\text{CDBM}} = 1.13$ and $EF5 = 0.96$ (we accept $EF5 = 1$). The result: E_{req} = 16.37 kWh/t.

2. Total specific energy consumption in WAM — CDBM circuit is found from Eq. (5): $E_{\text{total}} = 1.05{\cdot}16.37 = 17.19$ kWh/t.

3. The calculation of SEC for SAG milling from Eq. (6) disregards the correction factors since they are lower than 1 ($EF4_{\text{GDRM}} = 0.39$; $EF4_{\text{CDBM}} = 0.96$; $EF5 = 0.96$). The result: E_{WAM} = 15.01 kWh/t.

4. For the ball milling from Eq. (8): E_{CDBM} = 2.18 kWh/t.

5. The required power for the drives is found from Eqs. (9) and (10) at the output of 77 h/t: N_{WAM} = 1304 kWh/t; N_{CDBM} = 181 kWh/t.

At SAG stage, the required energy E_{req} = 1304 kW differs from the factual value of 1372 kW by 5.2%. For the ball milling, the described method gives E_{req} = 181 kW, while the calculated developed power of CDBM-2100×3000 is 165 kW, which makes the difference of 8.8%.

The estimated and factual data scatter is insignificant, which proves the proposed approach accuracy.

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