UDC 621.774.63:621.644.073 DOI: http://dx.doi.org/10.17580/cisisr.2017.02.06

SPRINGBACK COEFFICIENT OF THE MAIN PIPELINES' STEEL LARGE-DIAMETER PIPES UNDER ELASTOPLASTIC BENDING

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Key words:

large-diameter welded steel pipe, bending of pipe, bending moment, springback coefficient, elasto-plastic deformation, main gas-oil pipelines

ABSTRACT

The length of modern main gas-oil pipelines often reaches several thousand kilometers, and the pipelines are composed from an enormous number of steel large-diameter pipes. The production of the single-joint longitudinally welded steel pipes for main gas pipelines with a length of from 12 to 18 m, a diameter from 1020 to 1420 mm, a wall thickness from 19 to 48 mm from the steels of the strength classes from K38 to K65 is carried out in Russia at JSC "Vyksa Steel Works", JSC "Izhora Pipe Mill" and JSC "Chelyabinsk Pipe-Rolling Plant", as well as abroad at the pipe plants of the United States, Germany, China and India. During the ground laying of main pipelines in the trenches on the areas with a rugged terrain ("Yamal-Center", "Sakhalin-1", "Sakhalin-2", "Eastern Siberia - Pacific Ocean") or the submarine pipeline laying on the seabed ("Nord Stream", "Turkish Stream") the main large-diameter pipes experience the significant elastic and plastic bending deformations, which can lead to the industrial defects of main pipelines and the walls of main pipes. After the land and underwater laying, the main pipelines' pipes springback and fully or partially become straight, however, under an elastoplastic bending their residual longitudinal curvature is significantly different from zero. Under an excessive residual curvature, the main pipelines may have defects that are not compatible with the operational rules for pumping of gas and oil under a high-line pressure (from 1.2 to 10 MPa), or even to collapse. Disaster on the main pipelines often have the difficult removable environmental consequences on a global scale. Therefore, it is important to know the springback coefficient of the pipe during bending, allowing to calculate the residual curvature of the pipe after bending. In this paper we have obtained the springback coefficient of the pipe under bending for an elastoplastic medium with a linear hardening, depending on the diameter and the wall thickness of the pipe, the yield strength, the young's modulus and the hardening modulus of pipe's material. The research results can be used in the metallurgical and machine-building factories as well as in the construction of the gas-oil main pipelines.

1. Introduction

The steel bends from the steel thick-walled large-diameter pipes are widely used under laying the main gas-oil pipelines, as well as in the metallurgical and engineering industries [1–32].

The pipe-bend represents itself a single curved section of steel pipe, which is used to create the smooth changes of the pipeline systems' direction under the laying and repair of pipeline.

By types of manufacturing, the pipe-bends are divided into the bent, stamped-welded, cast, sectional-welded and steeply curved bends. The bent steel pipe-bends are most often used in the metallurgical, machine-building, petrochemical and natural gas industries, as well as in the construction of the main gas-oil pipelines.

In the construction of pipelines, it is necessary to change the directions of the main line, as many different obstacles often occur on the line's way, and that require a bypass of these obstacles with help of the pipe's twists.

Steel pipe-bend is a widespread product for connecting of two pipes with the same diameter. The connection of steel pipes and pipe-bends is made by welding. The typical bending angles of the pipe-bends are equal to 45°, 60°, 90° and 180°.

The popularity and demand of steel pipe-bends on the modern market of metal productions is due to the large number of different working mediums and substances that can be transported by means of pipeline constructions. The working medium for steel pipes and pipe-bends are the gas, petroleum, oil, water, non-aggressive and aggressive mediums, and various mixtures.

The ability to transport the specific fluid depends on the type of steel, from which the pipe line is made: the stainless steel is intended for more aggressive substances, but the carbon and low carbon steel is more suitable for non-aggressive transported media.

The diameter of steel pipe-bends for main pipelines can reach 1020–1420 mm and the wall thickness of pipe-bends can reach 48 mm [13–20]. Such pipe-bends are used in the main pipelines, transporting the working flows with the temperature from –70 °C up to + 450 °C and the pressure up to 16 MPa.

In the manufacture of the bent steel pipe-bends for main pipelines at the metallurgical enterprises, the original straight large-diameter steel pipe is bent several times on the three-roller mill (**fig. 1**) [19]. After the elastoplastic bending the pipe is partially springy back. For a high-quality manufacture of steel pipe-bends and an estimation of the final curvature's radius of the pipe-bends after bending, it is necessary to know the springback coefficient of the steel pipe under bending.

Modern production of steel pipe-bends take place on many large metallurgical plants, for example, at the Chelyabinsk Plant of Trubodetals "Magistral" and at

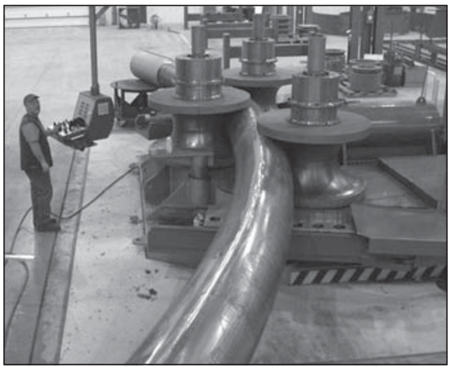

Fig. 1. Bending of steel pipe in the production of pipe-bends for main pipelines

the Chelyabinsk Plant "Trubodetal" (JSC "United Metallurgical Company").

The underwater sections of main gas-oil pipelines, as mentioned above, are also experiencing a significant power bending loads under the pipelines construction. There are two methods of bending pipes under a laying of main pipelines on the seabed – S-lay method and J-lay method (see **fig. 2**).

The bending of the pipes of the main pipeline "Turkish Stream" under its laying on the Black Sea bed (S-lay method) is shown in **fig. 3**.

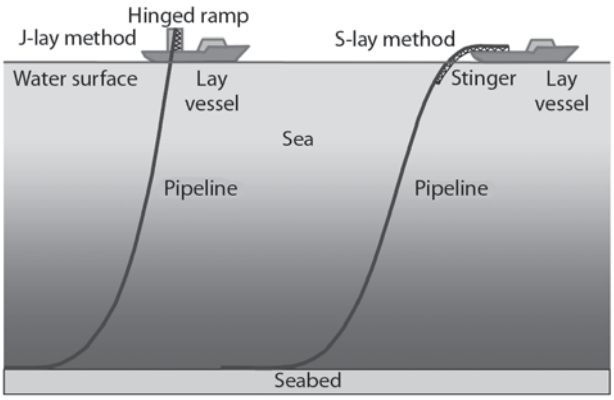

Fig. 2. The bending of pipes under S- and J- method of main pipeline's laying on the seabed

Fig. 3. The laying of the main pipeline "Turkish stream" on the Black Sea bed by means of a stinger (S-lay method)

Fig. 4. The pipes' bending of the main pipeline "Eastern Siberia – Pacific Ocean" under laying on the bottom of the trench

The bending of the pipes of the main pipeline "Eastern Siberia — Pacific Ocean" under its laying in the trenches on the open rugged terrain is shown in **fig. 4**.

2. Basic designations and assumptions

In the study of bending moment in the cross section of a circular pipe under bending, it is first necessary to consider the bending moment in the cross section of a circular beam under bending. The required bending moment of the pipe we receive as the difference between the bending moments of the beams, the diameters of which are respectively equal to the external and internal diameters of the pipe.

We consider a straight beam with a circular cross section of radius R. We assume that the longitudinal stretching of beam's material in the area of a plastic deformation has a linear hardening.

Let σ and ε be the normal stress and relative elongation of the beam in a tension; E, P and σ_y be the young's modulus, hardening modulus and yield stress of the beam's material.

Under the tension of beam in the region of elastic deformation, the normal stresses obey the Hooke's law

$$\sigma = E\varepsilon,$$

but in the area of hardening the dependence of a normal stress σ from a relative elongation ε has a straight form [13–20]

$$\sigma = \sigma_y + P\left(\varepsilon - \varepsilon_y\right), \quad \varepsilon_y = \frac{\sigma_y}{E}.$$

Next, we will consider only the bending of the beam. Under a beams' bending the maximum normal stresses are observed on the beam's surface at the points of the cross section which are most remote from the neutral plane of the beam.

The epure of normal stresses in the cross section of a round beam of the radius R under an elastoplastic bending is shown in **fig. 5**.

The cross section of the beam under bending is divided into two zones — an elastic zone and plastic zone. The

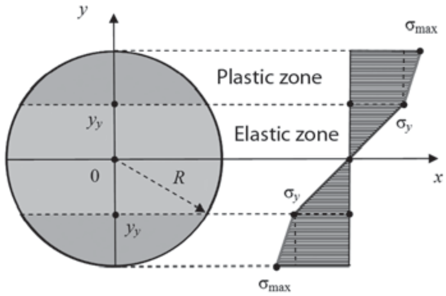

Fig. 5. The epure of the normal stresses in the cross section of round beam under bending

value of y_y, which defines the boundary of these zones, we find from the equations

$$\sigma_y = E\frac{y_y}{\rho}, \quad \sigma_y = E\varepsilon_y, \quad \varepsilon_y = \frac{\sigma_y}{E},$$

$$y_y = \frac{\sigma_y\rho}{E} = \frac{E\varepsilon_y\rho}{E} = \varepsilon_y\rho, \quad \frac{y_y}{R} = \frac{\sigma_y\rho}{ER}, \quad \frac{\sigma_y\rho}{Ey_y} = 1,$$

where ρ is the radius of the curvature of the beam's axis.

Under increasing of the bending moment and curvature, the elastic area of the beam are reduced, however, even at a very large curvature of the beam's neutral axis the elastic area of the beam does not disappear.

The radius of curvature of the longitudinal axis of a round beam with the cross-section's radius R, in which for the first time the normal tension on the beam's surface is equal to the yield strength σ_y, is equal to

$$\rho_y = E\frac{R}{\sigma_y}, \quad \sigma_y = E\frac{R}{\rho_y}, \quad \frac{\sigma_y\rho_y}{ER} = 1.$$

3. Bending moment of round beam

Under a pure elastic bending of a round beam, the bending moment M in the cross-section of a beam is equal to

$$M = \frac{\pi E R^4}{4\rho}.$$

We transform the expression for the bending moment under a pure elastic bending of a round beam to a dimensionless form

$$\frac{M}{\sigma_y R^3} = \frac{\pi}{4}\frac{ER}{\sigma_y\rho}.$$

Under a pure elastic bending of a round beam and $\rho = \rho_y = ER/\sigma_y$, we obtain

$$\frac{M}{\sigma_y R^3} = \frac{\pi}{4}.$$

Under an elastoplastic bending of a beam ($y_y/R = \sigma_y\rho/(ER) < 1$), the bending moment M in the cross-section of a round beam is equal to

$$M = -\frac{1}{6}\sigma_y R^3\left(1-\frac{P}{E}\right)\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)+$$

$$+\frac{1}{2}\left(1-\frac{P}{E}\right)\frac{ER^4}{\rho}\arcsin\left(\frac{\sigma_y\rho}{ER}\right)+\frac{\pi}{4}\frac{PR^4}{\rho}.$$

We transform the expression for the bending moment under the elastoplastic bending of a round beam to a dimensionless form

$$\frac{M}{\sigma_y R^3} = -\frac{1}{6}\left(1-\frac{P}{E}\right)\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)+$$

$$+\frac{1}{2}\left(1-\frac{P}{E}\right)\left(\frac{ER}{\sigma_y\rho}\right)\arcsin\left(\frac{\sigma_y\rho}{ER}\right)+\frac{\pi}{4}\frac{P}{E}\left(\frac{ER}{\sigma_y\rho}\right).$$

Under an elastoplastic bending of a round beam and $\rho = \rho_y = ER/\sigma_y$, we obtain

$$\frac{M}{\sigma_y R^3} = \frac{\pi}{4}.$$

4. Bending moment of pipe

Let h be the wall thickness of the pipe, R and r be the outer and inner radii of the pipe ($R = r + h$).

Under a pure elastic bending of a pipe ($y_y/R = \sigma_y\rho/ER \geq 1$), the bending moment M in the cross-section of a pipe is equal to

$$M = \frac{\pi E R^4}{4\rho} - \frac{\pi E r^4}{4\rho} = \frac{\pi E R^4}{4\rho}\left(1-\left(\frac{r}{R}\right)^4\right).$$

We transform the expression for the bending moment of a pipe under the pure elastic bending ($y_y/R = \sigma_y\rho/ER \geq 1$) to a dimensionless form

$$\frac{M}{\sigma_y R^3} = \frac{\pi}{4}\frac{ER}{\sigma_y\rho}\left(1-\left(\frac{r}{R}\right)^4\right).$$

Under a pure elastic bending of a pipe ($y_y/R = \sigma_y\rho/ER \geq 1$) and $\rho = \rho_y = ER/\sigma_y$, we obtain

$$\frac{M}{\sigma_y R^3} = \frac{\pi}{4}\left(1-\left(\frac{r}{R}\right)^4\right).$$

Under an elastoplastic bending of a pipe ($1 > y_y/R = \sigma_y\rho/ER \geq r/R$), the bending moment M in the cross-section of a pipe is equal to

$$M = -\frac{1}{6}\sigma_y R^3\left(1-\frac{P}{E}\right)\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)+$$

$$+\frac{1}{2}\left(1-\frac{P}{E}\right)\frac{ER^4}{\rho}\arcsin\left(\frac{\sigma_y\rho}{ER}\right)+\frac{\pi}{4}\frac{PR^4}{\rho}-\frac{\pi E r^4}{4\rho}.$$

We transform the expression for the bending moment of a pipe under the elastoplastic bending ($1 > y_y/R = \sigma_y\rho/ER \geq r/R$) to a dimensionless form

$$\frac{M}{\sigma_y R^3} = -\frac{1}{6}\left(1-\frac{P}{E}\right)\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)+$$

$$+\frac{1}{2}\left(1-\frac{P}{E}\right)\left(\frac{ER}{\sigma_y\rho}\right)\arcsin\left(\frac{\sigma_y\rho}{ER}\right)+\frac{\pi}{4}\left(\frac{ER}{\sigma_y\rho}\right)\left(\frac{P}{E}-\left(\frac{r}{R}\right)^4\right).$$

Under an elastoplastic bending of a pipe ($1 > y_y/R = \sigma_y\rho/ER \geq r/R$) and $\rho = \rho_y = ER/\sigma_y$, we obtain

$$\frac{M}{\sigma_y R^3} = \frac{\pi}{4}\left(1-\left(\frac{r}{R}\right)^4\right).$$

Under an elastoplastic bending of a pipe ($1 > y_y/R = \sigma_y\rho/ER \geq r/R$) and $\rho = \rho_{yr} = Er/\sigma_y$, we obtain

$$\frac{M}{\sigma_y R^3} = -\frac{1}{6}\left(1-\frac{P}{E}\right)\sqrt{1-\left(\frac{r}{R}\right)^2}\left(2\left(\frac{r}{R}\right)^2-5\right)+$$

$$+\frac{1}{2}\left(1-\frac{P}{E}\right)\left(\frac{R}{r}\right)\arcsin\left(\frac{r}{R}\right)+\frac{\pi}{4}\left(\frac{R}{r}\right)\left(\frac{P}{E}-\left(\frac{r}{R}\right)^4\right).$$

Under an elastoplastic bending of a pipe ($r/R > y_y/R = \sigma_y\rho/ER$), the bending moment M in the cross-section of a pipe is equal to

$$M = -\frac{1}{6}\sigma_y R^3\left(1-\frac{P}{E}\right)\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)+$$

$$+\frac{1}{2}\left(1-\frac{P}{E}\right)\frac{ER^4}{\rho}\arcsin\left(\frac{\sigma_y\rho}{ER}\right)+\frac{\pi}{4}\frac{PR^4}{\rho}+$$

$$+\frac{1}{6}\sigma_y r^3\left(1-\frac{P}{E}\right)\sqrt{1-\left(\frac{\sigma_y\rho}{Er}\right)^2}\left(2\left(\frac{\sigma_y\rho}{Er}\right)^2-5\right)-$$

$$-\frac{1}{2}\left(1-\frac{P}{E}\right)\frac{Er^4}{\rho}\arcsin\left(\frac{\sigma_y\rho}{Er}\right)-\frac{\pi}{4}\frac{Pr^4}{\rho}.$$

We transform the expression for the bending moment of a pipe under the elastoplastic bending ($r/R > y_y/R = \sigma_y\rho/ER$) to a dimensionless form

$$\frac{M}{\sigma_y R^3} = -\frac{1}{6}\left(1-\frac{P}{E}\right)\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)+$$

$$+\frac{1}{2}\left(1-\frac{P}{E}\right)\left(\frac{ER}{\sigma_y\rho}\right)\arcsin\left(\frac{\sigma_y\rho}{ER}\right)+\frac{\pi}{4}\frac{P}{E}\left(\frac{ER}{\sigma_y\rho}\right)\left(1-\left(\frac{r}{R}\right)^4\right)+$$

$$+\frac{1}{6}\left(1-\frac{P}{E}\right)\left(\frac{r}{R}\right)^3\sqrt{1-\left(\frac{R}{r}\right)^2\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{R}{r}\right)^2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)-$$

$$-\frac{1}{2}\left(1-\frac{P}{E}\right)\left(\frac{r}{R}\right)^4\left(\frac{ER}{\sigma_y\rho}\right)\arcsin\left(\frac{R}{r}\frac{\sigma_y\rho}{ER}\right).$$

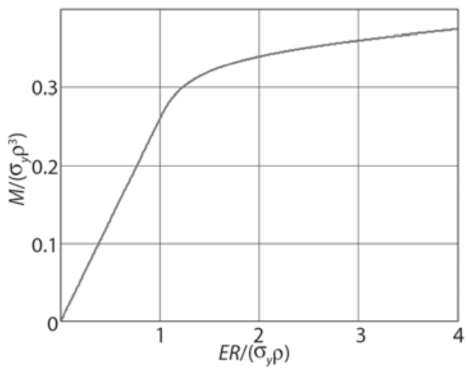

Fig. 6. The dependence of bending moment M from pipe's curvature $1/\rho$

Under an elastoplastic bending of a pipe ($r/R > y_y/R = \sigma_y\rho/ER$) and $\rho = \rho_{yr} = Er/\sigma_y$, we obtain

$$\frac{M}{\sigma_y R^3} = -\frac{1}{6}\left(1-\frac{P}{E}\right)\sqrt{1-\left(\frac{r}{R}\right)^2}\left(2\left(\frac{r}{R}\right)^2-5\right)+$$

$$+\frac{1}{2}\left(1-\frac{P}{E}\right)\left(\frac{R}{r}\right)\arcsin\left(\frac{r}{R}\right)+\frac{\pi}{4}\left(\frac{R}{r}\right)\left(\frac{P}{E}-\left(\frac{r}{R}\right)^4\right).$$

The dependence of $M/(\sigma_y R^3)$ from $ER/(\sigma_y\rho)$ for a pipe in $2R = 813$ мм, $h = 39$ мм, $P/E = 0{,}044$ (the parameters of the pipes for the underwater section of the gas pipeline "Turkish Stream") is shown in **fig. 6**.

5. Springback coefficient of round beam

In the basis of determining of the residual strains after the elastoplastic deformations, the Genki's theorem of unloading (1924 year) have a place [13–20]: the residual stresses are equal to the difference between the true stresses in the elasto-plastic body and the stresses which would be created in the body under the assumption of the ideal elasticity of the body's material.

Using the Genki's theorem of unloading, we obtain the equation for determining the residual radius of curvature ρ_{res} of the round beam:

$$\frac{M}{\sigma_y R^3} = \frac{\pi}{4}\frac{ER}{\sigma_y}\left(\frac{1}{\rho}-\frac{1}{\rho_{res}}\right),$$

$$\frac{1}{\rho_{res}} = \frac{1}{\rho}-\frac{4M}{\pi ER^4}.$$

The springback coefficient under bending of a round beam is equal to

$$\beta(\rho) = \frac{\rho_{res}}{\rho} = \frac{1}{1-\dfrac{4M\rho}{\pi ER^4}} = \frac{1}{1-\dfrac{4}{\pi}\dfrac{M}{\sigma_y R^3}\dfrac{\sigma_y\rho}{ER}},$$

$$\rho_{res} = \beta(\rho)\rho.$$

Under a pure elastic bending of a beam ($y_y/R = \sigma_y\rho/ER \geq 1$), the springback coefficient is equal to

$$\beta(\rho) = \infty.$$

Under an elastoplastic bending of a beam ($y_y/R = \sigma_y\rho/ER < 1$), the springback coefficient is equal to

$$\beta(\rho)=\frac{1}{\left[\left(1-\frac{P}{E}\right)\left[1+\frac{2}{3\pi}\frac{\sigma_y\rho}{ER}\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)-\frac{2}{\pi}\arcsin\left(\frac{\sigma_y\rho}{ER}\right)\right]\right]},$$

$$\beta(0)=\frac{1}{1-\frac{P}{E}},\quad \beta(\rho_y)=\infty.$$

Under an elastoplastic bending of a beam ($y_y/R = \sigma_y\rho/ER < 1$), for the Prandtl's diagram (the modulus hardening $P = 0$) we have

$$\beta(\rho)=\frac{1}{\left[1+\frac{2}{3\pi}\frac{\sigma_y\rho}{ER}\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)-\frac{2}{\pi}\arcsin\left(\frac{\sigma_y\rho}{ER}\right)\right]},$$

$\beta(0) = 1$, $\beta(\rho_y) = \infty$.

6. Springback coefficient of pipe

The equation for determining of the residual radius of pipe's curvature ρ_{res} has the form

$$\frac{M}{\sigma_y R^3}=\frac{\pi}{4}\frac{ER}{\sigma_y}\left(1-\left(\frac{r}{R}\right)^4\right)\left(\frac{1}{\rho}-\frac{1}{\rho_{\text{ост}}}\right),$$

$$\frac{1}{\rho_{res}}=\frac{1}{\rho}-\frac{4M}{\pi ER^4\left(1-\left(\frac{r}{R}\right)^4\right)}.$$

The springback coefficient under bending of a pipe is equal to

$$\beta(\rho)=\frac{\rho_{res}}{\rho}=\frac{1}{1-\frac{4M\rho}{\pi ER^4\left(1-\left(\frac{r}{R}\right)^4\right)}}=$$

$$=\frac{1}{1-\frac{4}{\pi}\frac{\left(\frac{M}{\sigma_y R^3}\right)\left(\frac{\sigma_y\rho}{ER}\right)}{\left(1-\left(\frac{r}{R}\right)^4\right)}},\quad \rho_{res}=\beta(\rho)\rho.$$

Under a pure elastic bending of a pipe ($y_y/R = \sigma_y\rho/ER \geq 1$), the springback coefficient is equal to

$$\beta(\rho) = \infty.$$

Under a elastoplastic bending of a pipe ($1 > y_y/R = \sigma_y\rho/ER \geq r/R$), the springback coefficient of a pipe under bending is equal to

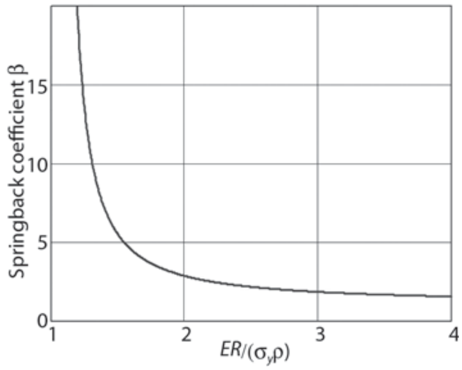

Fig. 7. The dependence of springback coefficient β from pipe's curvature 1/ρ

$$\beta(\rho)=\frac{1-\left(\frac{r}{R}\right)^4}{\left[\left(1-\frac{P}{E}\right)\left[1+\frac{2}{3\pi}\frac{\sigma_y\rho}{ER}\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)-\frac{2}{\pi}\arcsin\left(\frac{\sigma_y\rho}{ER}\right)\right]\right]},$$

$$\beta(\rho_y) = \infty.$$

Under an elastoplastic bending of a pipe ($1 > y_y/R = \sigma_y\rho/ER \geq r/R$), for the Prandtl's diagram (the modulus hardening $P = 0$) we have

$$\beta(\rho)=\frac{1-\left(\frac{r}{R}\right)^4}{\left[1+\frac{2}{3\pi}\frac{\sigma_y\rho}{ER}\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)-\frac{2}{\pi}\arcsin\left(\frac{\sigma_y\rho}{ER}\right)\right]},$$

$$\beta(\rho_y) = \infty$$

Under a elastoplastic bending of a pipe ($r/R > y_y/R = \sigma_y\rho/ER$), the springback coefficient of a pipe under bending is equal to

$$\beta(\rho)=\frac{1-\left(\frac{r}{R}\right)^4}{\left(1-\frac{P}{E}\right)\left[\begin{aligned}&1-\left(\frac{r}{R}\right)^4+\frac{2}{3\pi}\frac{\sigma_y\rho}{ER}\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2-5\right)-\\&-\frac{2}{\pi}\arcsin\left(\frac{\sigma_y\rho}{ER}\right)-\frac{2}{3\pi}\frac{\sigma_y\rho}{ER}\left(\frac{r}{R}\right)^3\times\\&\times\sqrt{1-\left(\frac{\sigma_y\rho}{ER}\right)^2\left(\frac{R}{r}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2\left(\frac{R}{r}\right)^2-5\right)+\\&+\frac{2}{\pi}\left(\frac{r}{R}\right)^4\arcsin\left(\frac{\sigma_y\rho}{ER}\frac{R}{r}\right)\end{aligned}\right]},$$

$$\beta(0) = \frac{1}{1 - \frac{P}{E}}.$$

Under an elastoplastic bending of a pipe ($r/R > y_y/R = \sigma_y\rho/ER$), for the Prandtl's diagram (the modulus hardening $P = 0$) we have

$$\beta(\rho) = \frac{1 - \left(\frac{r}{R}\right)^4}{\left[\begin{array}{l} 1 - \left(\frac{r}{R}\right)^4 + \frac{2}{3\pi}\frac{\sigma_y\rho}{ER}\sqrt{1 - \left(\frac{\sigma_y\rho}{ER}\right)^2}\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2 - 5\right) - \\ -\frac{2}{\pi}\arcsin\left(\frac{\sigma_y\rho}{ER}\right) - \frac{2}{3\pi}\frac{\sigma_y\rho}{ER}\left(\frac{r}{R}\right)^3\sqrt{1 - \left(\frac{\sigma_y\rho}{ER}\right)^2\left(\frac{R}{r}\right)^2} \times \\ \times\left(2\left(\frac{\sigma_y\rho}{ER}\right)^2\left(\frac{R}{r}\right)^2 - 5\right) + \frac{2}{\pi}\left(\frac{r}{R}\right)^4\arcsin\left(\frac{\sigma_y\rho}{ER}\frac{R}{r}\right) \end{array}\right]},$$

$$\beta(0) = 1.$$

The dependence of the springback coefficient β from $ER/(\sigma_y\rho)$ for a pipe in $2R = 813$ мм, $h = 39$ мм, $P/E = 0{,}044$ (the parameters of the pipes for the underwater section of the gas pipeline "Turkish Stream") is shown in **fig. 7**.

7. Conclusions

An analytical expressions for the springback coefficient of a pipe and a round beam under an elastoplastic bending are obtained. The research results can be applied in the metallurgical and machine-building industry under the manufacture of the pipes and pipe-bends, and the metal products made from a round beam and construction armatures [1–32].

REFERENCES

1. Calladine C. R. Plasticity for engineers. Theory and applications. Woodhead Publishing, 2000. 328 p.
2. Chakrabarty J. Theory of plasticity. Butterworth-Heinemann, 2006. 896 p.
3. Bhattacharyya D. Composite sheet forming. Vol. 11. Elsevier Science, 1997. 530 p.
4. Predeleanu M., Gilormini P. Advanced methods in materials processing defects. Vol. 45. Elsevier Science, 1997. 422 p.
5. Abe T., Tsuruta T. Advances in engineering plasticity and its applications (AEPA '96). Pergamon, 1996. 938 p.
6. Muhin U., Belskij S., Makarov E. Simulation of accelerated strip cooling on the hot rolling mill run-out roller table. *Frattura ed Integrita Strutturale*. 2016. Vol. 37. pp. 305–311.
7. Muhin U., Belskij S., Makarov E. Application of between-stand cooling in the production hot-rolled strips. *Frattura ed Integrita Strutturale*. 2016. Vol. 37. pp. 312–317.
8. Muhin U., Belskij S. Study of the influence between the strength of antibending of working rolls on the widening during hot rolling of thin sheet metal. *Frattura ed Integrita Strutturale*. 2016. Vol. 37. pp. 318–324.
9. Klocke F. Manufacturing processes 4. Forming. Springer, 2013. 516 p.
10. Kang S.-J. Sintering. Densification, grain growth and microstructure. Butterworth-Heinemann, 2004. 280 p.
11. Banabic D. Multiscale modeling in sheet metal forming. Springer, 2016. 405 p.
12. Hu J., Marciniak Z., Duncan J. Mechanics of Sheet Metal Forming. Butterworth-Heinemann, 2002. 211 p.
13. Shinkin V. N. The mathematical model of the thick steel sheet flattening on the twelve-roller sheet-straightening machine. Massage 1. Curvature of sheet. *CIS Iron and Steel Review*. 2016. Vol. 12. pp. 37–40.
14. Shinkin V. N. The mathematical model of the thick steel sheet flattening on the twelve-roller sheet-straightening machine. Massage 2. Forces and moments. *CIS Iron and Steel Review*. 2016. Vol. 12. pp. 40–44.
15. Shinkin V. N. Calculation of steel sheet's curvature for its flattening in the eight-roller straightening machine. *Chernye Metally*. 2017. No. 2. pp. 46–50.
16. Shinkin V. N. Calculation of bending moments of steel sheet and support reactions under flattening on the eight-roller straightening machine. *Chernye Metally*. 2017. No. 4. pp. 49–53.
17. Shinkin V. N. Calculation of technological parameters of O-forming press for manufacture of large-diameter steel pipes. *CIS Iron and Steel Review*. 2017. Vol. 13. pp. 33–37.
18. Shinkin V. N. Mathematical model of technological parameters' calculation of flanging press and the formation criterion of corrugation defect of steel sheet's edge. *CIS Iron and Steel Review*. 2017. Vol. 13. pp. 44–47.
19. Shinkin V. N. Asymmetric three-roller sheet-bending systems in steel-pipe production. *Steel in Translation*. 2017. Vol. 47. No. 4. pp. 235–240.
20. Shinkin V. N. Failure of large-diameter steel pipe with rolling scabs. *Steel in Translation*. 2017. Vol. 47. No. 6. pp. 363–368.
21. Lenard J. G. Metal Forming Science and Practice. Elsevier Science, 2002. 378 p.
22. Hingole R. S. Advances in metal forming. Expert system for metal forming. Springer, 2015. 116 p.
23. Qin Y. Micromanufacturing engineering and technology. William Andrew, 2015. 858 p.
24. Predeleanu M., Ghosh S. K. Materials processing defects. Vol. 43. Elsevier Science, 1995. 434 p.
25. Gorshkova A. L., Polulyakh L. A., Travyanov A. Ya., Dashevskii V. Ya., Yusfin Yu. S. Phosphorus distribution between phases in smelting high-carbon ferromanganese in the blast furnace. *Steel in Translation*. 2007. Vol. 37. No. 11. pp. 904–907.
26. Podgorodetskii G. S., Yusfin Yu. S., Sazhin A. Yu., Gorbunov V. B., Polulyakh L. A. Production of generator gas from solid fuels. *Steel in Translation*. 2015. Vol. 45. No. 6. pp. 395–402.
27. Orelkina O. A., Petelin A. L., Polulyakh L. A. Distribution of secondary gas emissions around steel plants. *Steel in Translation*. 2015. Vol. 45. No. 11. pp. 811–814.
28. Punin V. I., Kokhan L. S., Morozov Yu. A. Reduction of the length of strip rolled on roll-forming machines. *Metallurgist*. 2013. Vol. 56. Nos. 11–12. pp. 938–940.
29. Davim J. P. Materials Forming and Machining. Research and Development. Woodhead Publishing, 2015. 202 p.
30. Lin J., Balint D., Pietrzyk M. Microstructure evolution in metal forming processes. Woodhead Publishing, 2012. 416 p.
31. Rees D. Basic engineering plasticity. An introduction with engineering and manufacturing applications. Butterworth-Heinemann, 2006. 528 p.
32. Banabic D. Sheet metal forming processes. Constitutive modelling and numerical simulation. Springer, 2010. 301 p.