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INFLUENCE OF NON-LINEARITY OF HARDENING CURVE ON ELASTICOPLASTIC BEND OF RECTANGULAR ROD

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ABSTRACT

For the description of the hardening curve, they use the various classical approximations, for instance, the Prandtl's approximation (*Prandtl L.*), the linear approximation (*Smirnov-Alyaev G. A., Birger I. A., Moshnin E. N.*), the Nadai's approximation (*Nadai A. L.*) and the Ludwik's approximation (*Ludwik P.*). However, all these approximations have a large relative error (up to 35–45%) with respect to the experimental hardening curves obtained on the modern universal tensile machines. In addition, the significant drawback of the Nadai's and Ludwik's approximations is the infinite derivative (the tangent of inclination angle) of the stress curve at the beginning of the stretching of the rod ($\varepsilon = 0$). It contradicts the classical Hooke's law for the small elastic deformations and is not observed in none of the metals in practice. Therefore, below we will use a more accurate approximation by Shinkin V. N. in the form of a power series (a relative error 1–3 %). When metalware's bend of a by the external forces (the concentrated forces, distributed forces or pairs of forces), there are both positive and negative stresses in the metalware. In this case, one part of the metalware is stretched, and the second part is compressed. However, always in the center of the metalware there is a non-stresses surface (an axis), which bends, but has no stresses. The stresses, that arise during the bend of metalwares, create a bend's moment in its perpendicular plane, which tends to straighten the metalware after removing the external forces. The degree of straightening of the non-stresses surface after the bend is characterized by the spring-back factor. With the help of the spring-back factor we can find out the final form of the metalware after forces. In addition, after the elasticoplastic bend in the metalware the harmful stresses remain. Often the harmful stresses manifest themselves during the heating, cutting or secondary forming of the metalware. As a practical example of using of the proposed non-linear approximation method, the bend of the steel rectangular rod is considered, for which the bend moment, the spring-back factor and the residual stresses are calculated.

1. Non-linear approximation of the hardening curve

The plastic deformation of steels in the hardening zone is widely used in the production of metalwares from the steel rods, sheets and pipes [1–25].

Let the long rod with the initial perpendicular plane A_0 and the initial length l_0 be stretched by the longitudinal force F. Let l and Δl be the length and absolute lengthening of the rod under tension, then the relative elongation of the rod under tension $\varepsilon = \Delta l / l_0$. Let σ_y and σ_u be the yield strength and tensile strength of steel, and ε_y and ε_u be the relative elongation's values at which the yield strength and tensile strength are achieved.

The results of mechanical tests (the stretching diagram) of two flat specimens from the pipe steel of the strength class 485 are shown in **Fig. 1**. The length and width of the specimens, respectively, is 180 mm and 40.2 mm, the test temperature is +20 °C, the test procedure is ISO 6892.

The test results of specimens: the conventional yield strength $\sigma_{0.5}$ (0.5%) for specimens, respectively, equals 509 MPa and 517 MPa, the ultimate strength equals 609 MPa and 615 MPa, the relative elongation of specimens at the time of rupture equals 24% and 25% , the ratio of the yield strength to the ultimate strength equals 0.836 and 0.841.

From Fig. 1 it is clearly seen that the hardening curve in the stretching diagram is clearly nonlinear. The derivative (the tangent of an inclination angle) of the stress curve at the point, corresponding to the yield strength of the steel, has a bend and some finite (not infinite) value. In the hardening zone, the stress curve is a monotonically increasing convex curve, having a maximum (the zero derivative) at a point corresponding to the ultimate strength σ_u.

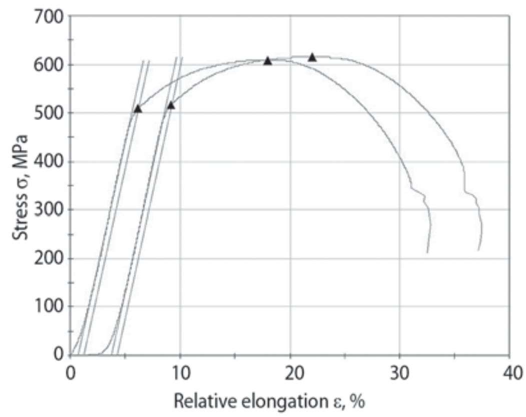

Fig. 1. The experimental stress-strain stretching diagram of two specimens from the pipe steel of strength class 485

The dependence of the stress on the relative elongation of the rod in the elastic zone obeys the classical linear Hooke's law (*Hooke R.*): $\sigma(\varepsilon) = E\varepsilon$ $(0 \le |\varepsilon| \le \varepsilon_y = \sigma_y / E)$, where $\sigma = F/A_0$ is a normal stress in the perpendicular plane of the steel rod, $\varepsilon = \Delta l / l_0$ is the relative elongation of the rod under tension, E is the young's modulus (the modulus of elasticity) of steel, σ_y is the yield strength of steel.

The direct non-linear Shinkin's approximation (*Shinkin V. N.*) for the hardening zone has the form

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$$\sigma(\varepsilon)=\begin{cases}E\varepsilon, & 0\le\varepsilon<\varepsilon_y=\dfrac{\sigma_y}{E};\\ \sigma_y+\sum\limits_n P_n\left[\left(\varepsilon-\dfrac{\sigma_y}{E}+\varepsilon_n\right)^n-\varepsilon_n{}^n\right], & \varepsilon\ge\varepsilon_y,\ \varepsilon_n\ge 0;\\ \sigma(\varepsilon)=-\sigma(-\varepsilon), & \varepsilon\le 0;\end{cases}$$

$$\sigma(\varepsilon_y)=\sigma_y,\quad \frac{d\sigma(\varepsilon_y)}{d\varepsilon}=\sum_n nP_n\varepsilon_n{}^{n-1}=P_y;$$

where P_y is the hardening module of steel at $\varepsilon=\varepsilon_y$, P_n are the coefficients of the series (may have both positive and negative values), ε_n are the displacements of the relative elongation ε relatively ε_y, n are the real numbers (can have both positive and negative values).

Under $P_n = 0$ we obtain the Prandtl's approximation. Under one term of the series, $\varepsilon_n = 0$ and $n = 1$, we obtain the linear approximation.

The peculiarities of the Shinkin's approximation are the introduction of the displacements ε_n (which allow us to have a finite value of the derivative of the hardening curve at the point corresponding to the yield strength) and the approximation of the hardening curve in the form of a finite or infinite series (which allows us to approximate the hardening curve from the yield strength to the ultimate strength with any precision).

Remark. Under the approximation of the whole hardening zone of steel (from the yield strength to the ultimate strength) by three members of the series and under the boundary conditions (the four boundary conditions)

$$\sigma(\varepsilon_y)=\sigma_y,\quad \sigma(\varepsilon_u)=\sigma_u,\quad \frac{d\sigma(\varepsilon_y)}{d\varepsilon}=P_y,\quad \frac{d\sigma(\varepsilon_u)}{d\varepsilon}=0;$$

the direct non-linear Shinkin's approximation is a cubic multinomial (polynomial) by ε and has the form

$$\sigma(\varepsilon)=\begin{cases}E\varepsilon, \quad 0\le\varepsilon<\varepsilon_y=\dfrac{\sigma_y}{E};\\ \sigma_y+P_y(\varepsilon-\varepsilon_y)-\dfrac{2P_y(\varepsilon_u-\varepsilon_y)-3(\sigma_u-\sigma_y)}{(\varepsilon_u-\varepsilon_y)^2}(\varepsilon-\varepsilon_y)^2+\\ +\dfrac{P_y(\varepsilon_u-\varepsilon_y)-2(\sigma_u-\sigma_y)}{(\varepsilon_u-\varepsilon_y)^3}(\varepsilon-\varepsilon_y)^3,\\ \varepsilon_y\le\varepsilon\le\varepsilon_u;\\ \sigma(\varepsilon)=-\sigma(-\varepsilon), \quad \varepsilon\le 0;\end{cases}$$

$$\sigma(\varepsilon_y)=\sigma_y,\quad \sigma(\varepsilon_u)=\sigma_u,\quad \frac{d\sigma(\varepsilon_y)}{d\varepsilon}=P_y,\quad \frac{d\sigma(\varepsilon_u)}{d\varepsilon}=0,$$

where σ_u is the ultimate strength of steel, ε_u is the relative elongation of the rod corresponding to the ultimate strength.

2. Bend moment at elasticoplastic bend of rectangular rod

Let h and b be, respectively, the thickness and width of the rectangular rod, and ρ be the radius of curvature of the non-stresses axis (the non-stresses surface) of the rod. The diagram of the normal stresses in the perpendicular plane of the rod at elasticoplastic bend is shown in **Fig. 2**, where the value [7, 14, 20] $y_y = \sigma_y\rho/E$ corresponds to the boundary between the elastic and plastic regions in the perpendicular plane of the rod.

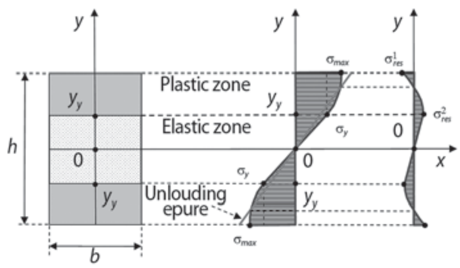

Fig. 2. The normal and residual stresses under the elasticoplastic bend of rod

The plastic deformation on the surface of a rectangular bar occurs when the curvature of the non-stresses axis of the bar [7, 14, 20] $\rho<\rho_{res}=Eh/(2\sigma_y)$. Under the bend of the rod, the normal stresses in the height of the perpendicular plane of the rod are equal to

$$\sigma(y)=\begin{cases}E\dfrac{y}{\rho}, & 0\le y<y_y=\dfrac{\sigma_y\rho}{E};\\ \sigma_y+\sum\limits_n\dfrac{P_n}{\rho^n}\left[\left(y-\dfrac{\sigma_y\rho}{E}+\varepsilon_n\rho\right)^n-\varepsilon_n{}^n\rho^n\right], & y_y\le y\le\dfrac{h}{2};\\ \sigma(y)=-\sigma(-y), & y\le 0;\end{cases}$$

$$\sigma(y_y)=\sigma_y,\quad \frac{d\sigma(y_y)}{dy}=\sum_n\frac{nP_n}{\rho^n}(\varepsilon_n\rho)^{n-1}=$$

$$=\frac{1}{\rho}\sum_n nP_n\varepsilon_n{}^{n-1}=\frac{P_y}{\rho}.$$

The bend moment in the rod's perpendicular plane [7, 14, 20] has the form

$$M=\frac{1}{4}bh^2\sigma_y\times$$

$$\times\left[\begin{aligned}&1-\frac{4}{3}\left(\frac{\sigma_y\rho}{Eh}\right)^2-\left(1-4\left(\frac{\sigma_y\rho}{Eh}\right)^2\right)\sum_n\frac{P_n}{\sigma_y}\varepsilon_n{}^n+\\&+\sum_n\frac{4}{(n+1)(n+2)}\frac{P_n}{\sigma_y}\left(\frac{h}{2\rho}\right)^n\times\\&\times\left\{\begin{aligned}&\left(1-2\frac{\sigma_y\rho}{Eh}+2\varepsilon_n\frac{\rho}{h}\right)^{n+1}\left(\frac{n+1}{2}+\frac{\sigma_y\rho}{Eh}-\varepsilon_n\frac{\rho}{h}\right)-\\&-\left(2\varepsilon_n\frac{\rho}{h}\right)^{n+1}\left((n+2)\frac{\sigma_y\rho}{Eh}-\varepsilon_n\frac{\rho}{h}\right)\end{aligned}\right\}\end{aligned}\right].$$

For the linear hardening ($n = 1$, $\varepsilon_n = 0$)

$$M=\frac{1}{4}bh^2\sigma_y\times$$

$$\times\left[1-\frac{4}{3}\left(\frac{\sigma_y\rho}{Eh}\right)^2+\frac{1}{3}\left(\frac{P}{E}\right)\left(\frac{Eh}{\sigma_y\rho}\right)\left(1-2\frac{\sigma_y\rho}{Eh}\right)^2\left(1+\frac{\sigma_y\rho}{Eh}\right)\right].$$

For the Prandtl's approximation ($n = 1$, $\varepsilon_n = 0$, $P_n = 0$)

$$M = \frac{1}{4} bh^2 \sigma_y \left[1 - \frac{4}{3} \left(\frac{\sigma_y \rho}{Eh} \right)^2 \right].$$

3. The spring-back factor at the bend of rectangular rod

After a removal of the external loads (forces), the rod straightens, but not to the end. The residual curvature of the non-stresses line of the rod is determined by the Hencky's theorem, according to which the difference between the true stresses and the imagined perfectly elastic stresses in the rod is equal to the residual stresses. In this case [6, 7, 14, 20]

$$\frac{1}{\rho_{res}} = \frac{1}{\rho} - \frac{12M}{Eh^3 b}, \quad \beta = \frac{\rho_{res}}{\rho} = \frac{1}{1 - \dfrac{12M\rho}{Eh^3 b}},$$

where ρ_{res} is the residual radius of curvature of the longitudinal non-stresses axis of the rod after its straightening (its spring back), β is the spring-back factor.

Substituting in this formula the value of the bend moment, we obtain

$$\beta = \frac{1}{1 - 3\dfrac{\sigma_y \rho}{Eh} \left[\begin{array}{l} 1 - \dfrac{4}{3}\left(\dfrac{\sigma_y\rho}{Eh}\right)^2 - \left(1 - 4\left(\dfrac{\sigma_y\rho}{Eh}\right)^2\right) \displaystyle\sum_n \dfrac{P_n}{\sigma_y} \varepsilon_n^{\,n} + \\ + \displaystyle\sum_n \dfrac{4}{(n+1)(n+2)} \dfrac{P_n}{\sigma_y} \left(\dfrac{h}{2\rho}\right)^n \times \\ \times \left\{ \begin{array}{l} \left(1 - 2\dfrac{\sigma_y\rho}{Eh} + 2\varepsilon_n \dfrac{\rho}{h}\right)^{n+1} \left(\dfrac{n+1}{2} + \dfrac{\sigma_y\rho}{Eh} - \varepsilon_n \dfrac{\rho}{h}\right) - \\ - \left(2\varepsilon_n \dfrac{\rho}{h}\right)^{n+1} \left((n+2)\dfrac{\sigma_y\rho}{Eh} - \varepsilon_n \dfrac{\rho}{h}\right) \end{array} \right\} \end{array} \right]}.$$

For the linear hardening ($n = 1$, $\varepsilon_n = 0$)

$$\beta = \frac{1}{\left(1 - \dfrac{P}{E}\right)\left(1 - 2\dfrac{\sigma_y\rho}{Eh}\right)^2 \left(1 + \dfrac{\sigma_y\rho}{Eh}\right)}.$$

For the Prandtl's approximation ($n = 1$, $\varepsilon_n = 0$, $P_n = 0$)

$$\beta = \frac{1}{\left(1 - 2\dfrac{\sigma_y\rho}{Eh}\right)^2 \left(1 + \dfrac{\sigma_y\rho}{Eh}\right)}.$$

4. Residual stresses at the bend of rectangular rod

According to the Hencky's theorem, the spring back of the rod takes place according to the linear law [6, 13, 14] $\sigma_{unl}(y) = \alpha_{unl} y$ (α_{unl} = const). The residual stresses are equal to the difference between the true stresses and the imagined perfectly elastic stresses (Fig. 2): $\sigma_{res} = \sigma - \sigma_{unl}$. The bend moments at loading and unloading are equal:

$$M = M_{unl} = \frac{1}{12} \alpha_{unl} bh^3, \quad \alpha_{unl} = \frac{12M}{bh^3},$$

$$\alpha_{unl} = 3\frac{\sigma_y}{h} \left[\begin{array}{l} 1 - \dfrac{4}{3}\left(\dfrac{\sigma_y\rho}{Eh}\right)^2 - \left(1 - 4\left(\dfrac{\sigma_y\rho}{Eh}\right)^2\right) \displaystyle\sum_n \dfrac{P_n}{\sigma_y} \varepsilon_n^{\,n} + \\ + \displaystyle\sum_n \dfrac{4}{(n+1)(n+2)} \dfrac{P_n}{\sigma_y} \left(\dfrac{h}{2\rho}\right)^n \times \\ \times \left\{ \begin{array}{l} \left(1 - 2\dfrac{\sigma_y\rho}{Eh} + 2\varepsilon_n \dfrac{\rho}{h}\right)^{n+1} \left(\dfrac{n+1}{2} + \dfrac{\sigma_y\rho}{Eh} - \varepsilon_n \dfrac{\rho}{h}\right) - \\ - \left(2\varepsilon_n \dfrac{\rho}{h}\right)^{n+1} \left((n+2)\dfrac{\sigma_y\rho}{Eh} - \varepsilon_n \dfrac{\rho}{h}\right) \end{array} \right\} \end{array} \right].$$

For the linear hardening ($n = 1$, $\varepsilon_n = 0$)

$$\alpha_{unl} = 3\frac{\sigma_y}{h} - 4\frac{\sigma_y}{h}\left(\frac{\sigma_y\rho}{Eh}\right)^2 + \frac{P}{\rho}\left(1 - 2\frac{\sigma_y\rho}{Eh}\right)^2 \left(1 + \frac{\sigma_y\rho}{Eh}\right).$$

For the Prandtl's approximation ($n = 1$, $\varepsilon_n = 0$, $P_n = 0$)

$$\alpha_{unl} = 3\frac{\sigma_y}{h} - 4\frac{\sigma_y}{h}\left(\frac{\sigma_y\rho}{Eh}\right)^2.$$

The minimum residual stress is observed on the surface of the rod, is less than zero and equal to

$$\sigma_{res}^{\,1} = \sigma_y + \sum_n \frac{P_n}{\rho^n}\left[\left(\frac{h}{2} - \frac{\sigma_y\rho}{E} + \varepsilon_n\rho\right)^n - \varepsilon_n^{\,n}\rho^n\right] -$$

$$-\frac{3}{2}\sigma_y \left[\begin{array}{l} 1 - \dfrac{4}{3}\left(\dfrac{\sigma_y\rho}{Eh}\right)^2 - \left(1 - 4\left(\dfrac{\sigma_y\rho}{Eh}\right)^2\right) \displaystyle\sum_n \dfrac{P_n}{\sigma_y} \varepsilon_n^{\,n} + \\ + \displaystyle\sum_n \dfrac{4}{(n+1)(n+2)} \dfrac{P_n}{\sigma_y} \left(\dfrac{h}{2\rho}\right)^n \times \\ \times \left\{ \begin{array}{l} \left(1 - 2\dfrac{\sigma_y\rho}{Eh} + 2\varepsilon_n \dfrac{\rho}{h}\right)^{n+1} \left(\dfrac{n+1}{2} + \dfrac{\sigma_y\rho}{Eh} - \varepsilon_n \dfrac{\rho}{h}\right) - \\ - \left(2\varepsilon_n \dfrac{\rho}{h}\right)^{n+1} \left((n+2)\dfrac{\sigma_y\rho}{Eh} - \varepsilon_n \dfrac{\rho}{h}\right) \end{array} \right\} \end{array} \right].$$

For the linear hardening ($n = 1$, $\varepsilon_n = 0$)

$$\sigma_{res}^{\,1} = -\frac{1}{2}\sigma_y\left(1 - \frac{P}{E}\right)\left(1 - 4\left(\frac{\sigma_y\rho}{Eh}\right)^2\right) < 0.$$

For the Prandtl's approximation ($n = 1$, $\varepsilon_n = 0$, $P_n = 0$)

$$\sigma_{res}^{\,1} = -\frac{1}{2}\sigma_y\left(1 - 4\left(\frac{\sigma_y\rho}{Eh}\right)^2\right).$$

The maximum residual stress is observed at $y = y_y$, is greater than zero and equal to

$$\sigma_{res}^{\,2} = \sigma_y -$$

$$-3\sigma_y \frac{\sigma_y\rho}{Eh} \left[\begin{array}{l} 1 - \dfrac{4}{3}\left(\dfrac{\sigma_y\rho}{Eh}\right)^2 - \left(1 - 4\left(\dfrac{\sigma_y\rho}{Eh}\right)^2\right) \displaystyle\sum_n \dfrac{P_n}{\sigma_y} \varepsilon_n^{\,n} + \\ + \displaystyle\sum_n \dfrac{4}{(n+1)(n+2)} \dfrac{P_n}{\sigma_y} \left(\dfrac{h}{2\rho}\right)^n \times \\ \times \left\{ \begin{array}{l} \left(1 - 2\dfrac{\sigma_y\rho}{Eh} + 2\varepsilon_n \dfrac{\rho}{h}\right)^{n+1} \left(\dfrac{n+1}{2} + \dfrac{\sigma_y\rho}{Eh} - \varepsilon_n \dfrac{\rho}{h}\right) - \\ - \left(2\varepsilon_n \dfrac{\rho}{h}\right)^{n+1} \left((n+2)\dfrac{\sigma_y\rho}{Eh} - \varepsilon_n \dfrac{\rho}{h}\right) \end{array} \right\} \end{array} \right].$$

For the linear hardening ($n = 1$, $\varepsilon_n = 0$)

$$\sigma_{res}{}^2 = \sigma_y\left(1-\frac{P}{E}\right)\left(1-2\frac{\sigma_y\rho}{Eh}\right)^2\left(1+\frac{\sigma_y\rho}{Eh}\right) > 0.$$

For the Prandtl's approximation ($n = 1$, $\varepsilon_n = 0$, $P_n = 0$)

$$\sigma_{res}{}^2 = \sigma_y\left(1-2\frac{\sigma_y\rho}{Eh}\right)^2\left(1+\frac{\sigma_y\rho}{Eh}\right) > 0.$$

5. Numerical calculations

Consider the approximation of the hardening zone of two steel rods, the stretching diagram of which is shown in Fig. 1. In this case, the hardening modulus at $\varepsilon = \varepsilon_y$ is equal to P_y = 2155 MPa, the average yield strength is σ_y = (509 + 517)/2 = 513 MPa, the average ultimate strength is σ_u = (609 + 615)/2 = 612 MPa, the average value of the relative elongation (corresponding to the ultimate strength σ_u) is ε_u = 0.125. The experimental hardening curve (Fig. 1) and the curve of the direct approximation by Shinkin for the hardening zone are shown in **Fig. 3**.

Consider the bend of the rectangular rod, the mechanical characteristics of the steel which correspond to Fig. 1. Let the young's modulus of steel $E = 2\times10^5$ MPa, the rod's thickness h = 0.02 m, the rod's width b = 1.8 m, and the radius of curvature of the non-stresses surface of the rod ρ = 0.25 m. Then the bend moment of the perpendicular plane of the rectangular rod, the spring-back factor and the residual stresses are respectively equal to

$$M = 99.89\ \text{kPa}\cdot\text{m}^3,\quad \beta = 1.116,$$
$$\sigma_{res}{}^1 = -257.9\ \text{MPa},\quad \sigma_{res}{}^2 = 459.6\ \text{MPa}.$$

6. Conclusions

Under the elasticoplastic bend of steel sheet and rectangular rod (with taking into account the nonlinearity of the hardening curve), the analytical expressions for the bend moment, the spring-back factor and the residual stresses are obtained.

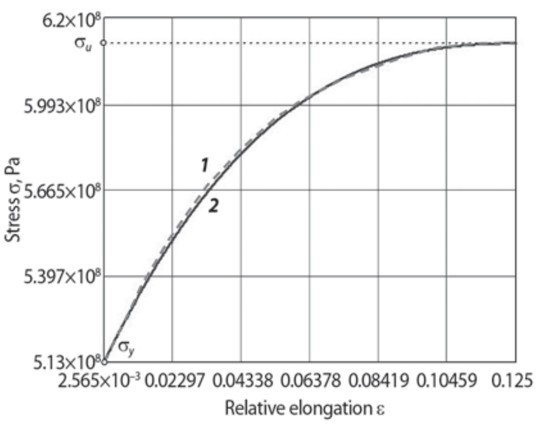

Fig. 3. The stress curves in the hardening zone of steel: the experimental dotted curve (*1*) and the curve of the direct approximations by Shinkin (*2*); $\varepsilon_y \le \varepsilon \le \varepsilon_u$

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