Evaluation of the convexity of the transverse profile of rolled steel strip

D. L. Shalaevsky, *Cand. Eng., Associate Prof., Dept. of Metallurgy, Mechanical Engineering and Technological Equipment*[1]*, e-mail: shal-dmitrij@yandex.ru*

[1] *Cherepovets State University, Cherepovets, Russia*

Uneven elastic deformations of the work roll barrel surfaces in a quarto rolling mill stand are one of the main factors determining the transverse profile of the finished rolled steel strip and its flatness characteristics. Known factors that determine the cross-section of the rolled product include work roll deflection due to rolling forces and forces of additional bending systems, uneven thermal expansion, and work roll barrel surface wear. The most significant factor contributing most to the formation of the rolled product's transverse profile is the elastic deformation of the roll system. The most common methods for calculating roll deflection, which are used in roll barrel contouring, are based on the Castigliano theorem; this theorem defines the required parameter as a partial derivative of the potential deformation energy due to this deformation presented by contact pressure in the deformation area, counterbending forces, or additional bending forces. It should be also noted that most known methods do not take into account the uneven distribution of contact stresses in the deformation area, caused, among other things, by transverse thickness deviation in the rolled product. To account for the latter noted parameter, this study proposes an approach to calculating the elastic deformations of the working rolls of a quarto rolling mill using the initial parameter method, taking into account all applicable loads. The results presented in the study demonstrate that the transverse profile of rolled semiproduct has very little effect on the cross-section profile of the finished rolled product.

It is concluded that there is no need to consider the cross-section profile of the rolled steel at the rolling stand entrance when evaluating the cross-section profile of the rolled steel strip. The paper also presents the results of calculating the cross-section convexity of the rolled steel strip and compares the obtained characteristics with their calculated values. The subject of this study is the cross-section convexity of the rolled steel strip. The proposed method allows the reliable calculation of the cross-section profile of the finished rolled steel strip. Its accuracy is no lower than that of known approaches, but the calculation complexity is significantly lower. The objective of this study is to develop a reliable and least labor-intensive method for calculating the cross-section convexity of flat rolled steel. The results can be used in the development of roll contouring for sheet rolling mills.

Key words: rolled steel strip, quarto rolling stand, elastic deformation of a roll, strip transverse profile, contact stress, additional roll bending force.

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Introduction

Elastic deformation of working and back-up rolls is the main factor determining transverse profile of a finished strip. Rolling forces, counterbending forces and additional bending forces as well as bearing reaction forces are considered as the force and technological parameters having the effect on rolls deflection. Accuracy of deformation calculation will be determined by reliability of rolling force and back-up rolls reaction calculation techniques, as well as the methods for evaluation of these elastic deformations. The following methods for evaluation of elastic deformation value are known: the Castigliano theorem, the Mohr integral and the numerical solution of a differential equation for beam bended axis (the method of initial parameters). The problem of rolled semiproduct cross-section influence on finished rolled product cross-section is also considered as an interesting one.

The above-mentioned technique, which allows calculating of rolls deformation and is based on practical use of the Castigliano theorem, is presented in the work [1]. This method can be used for calculation of working rolls deflection in a duo rolling stand. To evaluate deformations in a quarto rolling stand, the method based on practical use of the Castigliano theorem was suggested in the work [2]. This approach takes into account possible action of additional bending forces and counterbending forces in working rolls. Sufficient labour intensity of this approach during its use and absence of practical statistical data about its authenticity can be noted.

The numerical finite element method, which was realized in CAD systems, was sufficiently distributed for calculation of elastic deformations in rolling rolls. The results of its use are presented in the works [3-5]. A serious labour intensity can be considered as a deficiency of this method, because it is required to set the new boundary conditions for each loading condition of a roll system, and calculation time can be essential. The calculation technique for elastic deformations using elasticity module of a rolling stand is presented in the research [6].

Uneven thermal profile of rolling roll barrels and wear of their surface also have the effect on rolled product cross-section profile, in addition to elastic deflection of rolls. To evaluate the above-mentioned thermal profile, the technique for thermal calculation of wide hot stril mill rolls can be used; it is based on thermal balance equations of working and back-up rolls, taking into account their mutual heat exchange as well as heat exchange with a strip, cooling water and the environment [7]. Accuracy of this model will be essentially determined by the values of thermal exchange coefficients, which in their turn are calculated via empiric formulas. Low accuracy of pauses accounting in rolling schedule can be noted as a deficiency if this model. The simplified technique for calculation of working rolls thermal profile in a hot rolling mill was examined in the research [8].

The lecture [9] considers the problem of temperature lowering in a rolling rolls barrel to an acceptable level with use of construction varying for cooling system collectors. However, the effect of thermal conditions in rolling rolls on strip transverse profile was not studied.

The technique is based on accurate solution of the heat conductivity equation and measuring the temperature distribution along working rolls barrel. The report [10] presents an experience of varying construction of cooling system collectors in order to decrease rolling rolls barrel temperature to an acceptable level. Calculating experience displayed that the results of the above-mentioned work [10] can be used for prediction of thermal profile of working rolls barrels. These results which were adapted for rolling stands conditions of a concrete rolling mill, taking into account real duration of pauses during rolling.

The results of numerical simulation of a strip temperature conditions are presented in the work [11]. This study was carried out in a stationary mode. Working rolls were presented as two hollow cylinders which obtain heat die to the contact with a billet and which are cooled due to convection at its external surface and axial hole surface. Dependence between roll thermal behaviour (from one side) and cylinder rotation speed and heat exchange conditions with the environment (from other side) is determined in this work.

The effect of thermal conditions of rolling rolls on transverse profile and flatness of hot-rolled steel strips is examined in the researches [12, 13]. Wear of roll barrel surfaces is one more factor having the effect of cross-section profile of rolled products. The work [14] presents a mathematical model of forming a hot-rolled strip transverse profile at a wide hot strip mill; it takes into account grinding profile and bending of rolls, their elastic flattening in the contact with strip and temperature expanding, as well as surface wear of rolling roll during rolling.

It is mentioned that such model was used at the wide hot strip mill 2000 for improvement of the hot rolling technology and optimization of rolling process planning for electrical steels with high requirements to accuracy of a cross-section profile. Mathematical model of roll barrel wear at hot-rolling mill, which is based on the experimental data, is presented in the work [15].

The main task of the research [16] is determination of uneven roll barrel wear via statistical analysis in order to reveal the cause-and-effect relationships. It is testified that deviation between the calculation results and the actual values did not exceed 3 %, what allowed to develop recommendations for varying the values of technological parameters, which compensate negative effect of back-up rolls wear in manufacture of rolled products. Symmetric and asymmetric rolling processes were examined in the work [17]. The Arhard equation was used for roll wear determination. The main regularities of roll barrel surface wear depending on rolling parameters were established.

The research [18] examined working roll barrel wear as one of the causes of strip surface contamination. It was established that increase of strip reduction and its material strengthening, as well as increase of the temperature of lubricating and cooling fluid, rises working roll barrel wear. The problem of roll wear in asymmetric cold rolling is considered in the work [19]. The monograph [20] is devoted to the problems of resistance increase for rolling rolls; it is based on the suggestion that rolling temperature, pressure in a deformation area, reduction, hardness of roll surface layers are considered as the main causes of increased wear.

The results of roll wear examination and its influence on strip transversal profile for wide hot strip mill 2000 of “Severstal” JSC are presented in [21]. The technique of carrying-out the research and used special devices are described. The profiles of roll barrel wear and wear dynamics of back-up rolls were determined experimentally depending on the amount of rolled metal. The obtained relationships also can be used in prediction of the profile of finished strip. Use of rolling process parameters in forming a transverse profile of finished strip is presented in the work [22].

The operating conditions of rolling mill rolls were examined in the research [23], the main causes of their barrel wear and corresponding variation of transverse profile of finished strips were established. The work [24] presents the relationship between transverse profile of finished strips and its flatness. Based on the above-described researches, it is clear that the problem of reliable prediction transverse profile of rolled strips, that is the theme of this work, is rather actual. So, development of the reliable and minimal labour-intensive calculation technique for convexity of flat rolled steel cross-section is the aim of this research.

The following tasks were solved in this work:

– checking of presence of the effect of transverse thickness deviation on transverse profile of a finished strip;

– determination of a simple and reliable method for evaluation of transverse convexity of finished product, providing sufficient calculating accuracy.

Goal setting

To calculate elastic deformation of rolling rolls, the initial parameter method can be used. It is known from the course of strength of materials, taking into account unequal reduction and contact pressure values along the strip width, which are caused by transverse thickness deviation of a rolled metal. This method is rather simple, it does not require separate evaluation of forces and pressures, applied by a working roll. In correspondence to this method, deflection of axes of back-up and working rolls f can be found out from the following expression:

$$EIf = EI\vartheta_0 z + EIf_0 \pm \sum_{i=1}^{n} \frac{M_i c^{\,2}}{2!} \pm \sum_{j=1}^{m} \frac{P_j c^{\,3}}{3!} \pm \sum_{k=1}^{l} \frac{q_k c^4}{4!} \pm \sum_{k=1}^{l} \frac{q_k' c^5}{5!} \quad (1)$$

where EI – rigidity of back-up roll cross-section; f – deflection of back-up roll axis in the examined point along the roll length; f_0 – deflection of roll axis in the ultimate left point; υ_0 – turning angle of the cross-section в in the ultimate left point of the calculation scheme; z – coordinate of the point, where deflection is determined; M – a torque located to the left from the point, where deflection is determined (c – distances from the point of torque applying to the point, where deflection is determined); P – force located to the left from the point, where deflection is determined (c – distances from the point of force applying to the point, where deflection is determined), q – distributed load, located to the left from the point, where deflection is determine (c – distances from the point of distributed load applying to the point, where deflection is determined), q' – the value of distributed load, opposite directed and equal by a module q, applied from the point, where action of applied force q is terminated (c – distance from the point of starting of distributed force q' applying to the point, where deflection is determined); n, m, l – number of torques of external forces, concentrated external forces and distributed external loads, having the effect on the calculation scheme, respectively; i, j, k – sequential number for external force torque, concentrated external force torque and distributed external load torque.

To take into account the effect of transverse thickness deviation, elastic deflection of back-up and working rolls was determined in each point of its barrel length, in the conditions of variable contact pressure. This contact pressure was determined in the following way:

– contact stress was calculated using the known technique, based on previously measured thickness of rolled semiproduct in the preset point and required thickness of rolled product;

– elastic deflection of back-up and working rolls in each point was evaluated via differential equation of a beam bending axis, e.g. via the initial parameters method as a numerical method for solving this equation;

– elastic flattening of back-up and working rolls in their contact was determined via the known formula [25];

– strip transverse profile was revealed based on determined values of elastic deformation and elastic flattening, taking into account thermal profile of rolls, which can be calculated using the balanced model [25];

– the gap shape between rolling rolls was determined as a resulting initial contouring, elastic deformation, thermal expanding and roll barrel surface wear;

– strip reduction was determined in details in each point along its width, using the previously revealed gap between rolling rolls;

– this pressure was compared with previously determined value: if these values didn't coincide, elastic deformations we recalculated with use of the newly found value of this pressure, and then calculation and comparison operations for contact stresses were repeated.

The scheme for calculation of working roll elastic deflection, taking into account various specific pressure values along a roll barrel width and based on the example of a rolling stand at the wide hot strip mill 2000, is presented in the **Fig. 1**.

If we compose the equation of initial parameters along the length of a contact zone between the barrels of back-up and working rolls from the left support to the middle of a roll barrel, with neglecting the found rather small values of initial parameters f_0 and υ_0, we shall obtain the following equation for evaluation of back-up roll axis deflection in its middle:

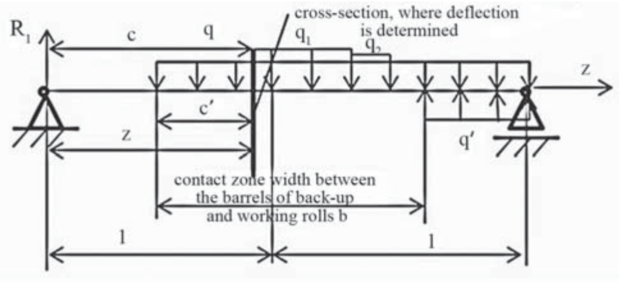

Fig. 1. Calculating scheme for determination of back-up roll axis deflection (R_1 – reaction of the left bearing link)

$$f_{\text{b-up}} = \frac{1}{EY} \cdot \left(\frac{R_1 \cdot c^3}{3!} - \frac{q \cdot \left(c - [0{,}5 \cdot l - 0{,}5 \cdot b]\right)^4}{4!} \right) \quad (2)$$

where l – back-up roll barrel length; R_1 – reaction in the back-up roll bearing: $R_1 = P/2 + P_{\text{ben}}$ (P_{ben} – counterbending force for a working roll).

Fictitious load q' is absent in the equation (2), because the contact between back-up and working rolls at the wide hot strip mill 2000 is realized along their complete width (we can neglect bevels on a back-up roll barrel).

To take into account the effect of transverse thickness deviation and various reductions along strip width in calculation of deflection in other areas with action of distributed load q_1, q_2 etc., we shall add the following expression to the equation (2):

$$\frac{\Delta q \cdot \left(c - \left[\dfrac{l-b}{2} \right] \right)^4}{4!}$$

where Δq – increase of specific load due to varying of rolled semiproduct thickness: $\Delta q = q_i - q_{i-1}$.

Calculation for equal reduction and contact pressure values along strip width is similar; however, we shall use the same specific pressure, without its increase Δq.

In addition to elastic deflection of a back-up roll, elastic deflection of a working roll will occur under rolling force action from strip, force action from a back-up roll and counterbending force P_{ben}, which is applied to rolling roll necks. The scheme of a working roll deformation calculation on the example of a rolling stand in the finished group of a wide hot strip mill 2000 is presented in the **Fig. 2**.

In the same way, let us compose the equation of working roll elastic deflection along its width:

$$f_w = \frac{1}{EY} \cdot \left(\frac{P_{ben} \cdot c^3}{3!} - \frac{q \cdot c^4}{4!} + \frac{q \cdot (c - 0.5 \cdot l + 0.5 \cdot b)}{4!} \right) \quad (3)$$

where , $q = \frac{P + P_{ben}}{l}$, $q" = \frac{P}{b}$, – counterbending force.

The complete deformation of a working roll in each point of its surface was calculated as a sum of deformations of working f_w and back-up $f_{\text{b-up}}$ rolls in this point, in corresponding direction:

$$f = f_{\text{w}} + f_{\text{b-up}}. \tag{4}$$

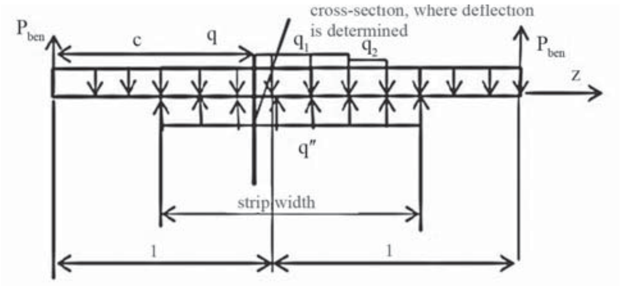

Fig. 2. The scheme for calculation of a working roll elastic deformation

Results and discussion

Based on the calculation results for a cross-section profile of a steel strip using the proposed approach (taking into account uneven strip reduction along its width and not taking into account this uneven reduction), the values of cross-section convexity of rolled product with thickness 3 mm and width 1,000 mm are presented; rolling process was carried out in the finishing group of the wide hot strip rolling mill 2000 for 09G2S steel. These results are displayed in the **Table**.

The results of cross-section convexity calculation for rolled steel strip showed absence of the effect of semiproduct convexity on this cross-section, and, respectively, semiproduct cross-section; it can be confirmed by the data from the Table. According to these data, the strip cross-section convexity are not identified as inherited ones, what stipulates possibility of appearance of flat defects. Such conclusion can be made on the base of well-known works devoted to rolling of flat steel strips [2, 26]. It was concluded that forming of flatness defect of "wave" type with amplitude up to 30 mm is possible in accordance with a flat defect model forming [26, 27].

In order to evaluate reliability of the newly proposed calculating method, it was conducted using calculation of the cross-section profile for hot-rolled strip at the exit from the finishing stand of wide hot strip rolling mill 2000, and afterwards the obtained results were compared with the experimental data. Thermal profile of a roll barrel surface was taken into account, as well as its initial contouring and current wear.

The results of such comparison are presented in the **Fig. 3**. Checking of its reliability demonstrates possibility of the newly proposed calculating method of the newly proposed calculating method to predict almost 70 % of cross-section profile variability, what is a sufficient value, taking into account that the used values of actual strip convexity are average ones along the whole rolled product length.

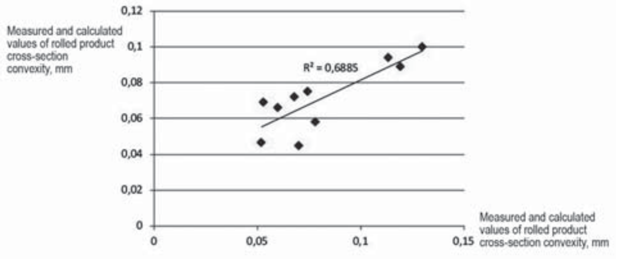

Fig. 3. Relationship between measured and calculated values of rolled product cross-section convexity

The errors were evaluated via two methods: the newly proposed in this research and the presented in the monograph [2]. The calculating histograms are displayed in the **Fig. 4**. To build these histograms, the data about manufacture of

The results of cross-section convexity calculation for rolled steel strip

No.	Calculating variant	Profile convexity of rolled semiproduct in the rolling stand No., mm							
		No. 6	No. 7	No. 8	No. 9	No. 10	No. 11	No. 12	No. 12 actual
1	Taking into account uneven strip reduction along its width	0.043	0.049	0.055	0.061	0.068	0.071	0.078	0.084
2	Not taking into account uneven strip reduction along its width	0.042	0.047	0.052	0.058	0.066	0.069	0.075	

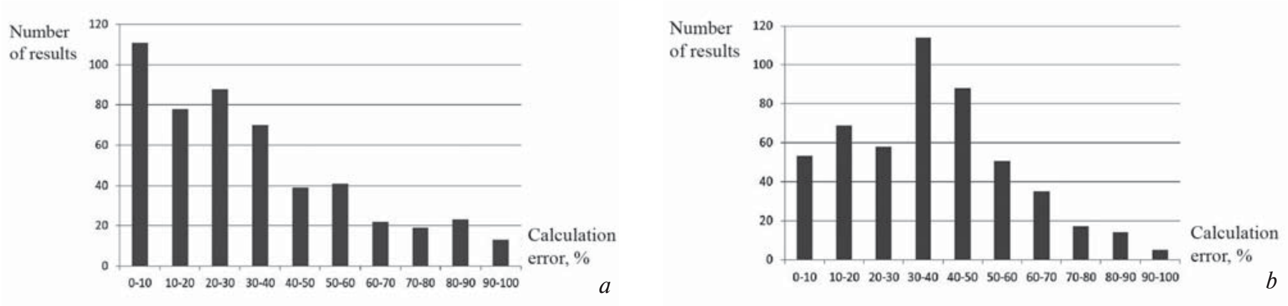

Fig. 4. The histogram of calculation error for rolled product cross-section convexity using the newly proposed (*a*) and well-known (*b*) [2] techniques

1000 hot-rolled strips in the finishing stand group of the wide hot strip rolling mill 2000 were used. Their grade and dimension ranges were as follows: steels 08ps, 09G2S, St3kp with strip thickness 1.5–5.0 mm and strip width 1000–1525 mm.

It can be seen from the presented diagrams, that the newly proposed technique provides calculation with an average error value 30–40 %. In this case, half of all calculations via this technique and via well-known technique are characterized by an error smaller than 40 %, while the approach described in this research has smaller number of results with an error larger than 70 %.

Conclusion

Uneven elastic deformations of the work roll barrel surfaces in a quarto rolling mill stand are one of the main factors determining the transverse profile of the finished rolled steel strip and its flatness characteristics. Known factors that determine the cross-section of the rolled product include work roll deflection due to rolling forces and forces of additional bending systems, uneven thermal expansion, and work roll barrel surface wear. The most significant factor contributing most to the formation of the rolled product's transverse profile is the elastic deformation of the roll system.

The most common methods for calculating roll deflection, which are used in roll barrel contouring, are based on the Castigliano theorem; this theorem defines the required parameter as a partial derivative of the potential deformation energy due to this deformation presented by contact pressure in the deformation area, counterbending forces, or additional bending forces.

The results presented in the study demonstrate that the transverse profile of rolled semiproduct has very little effect on the cross-section profile of the finished rolled product. It is concluded that there is no need to consider the cross-section profile of the rolled steel at the rolling stand entrance when evaluating the cross-section profile of the rolled steel strip. The paper also presents the results of calculating the cross-section convexity of the rolled steel strip and compares the obtained characteristics with their calculated values.

The proposed method allows the reliable calculation of the cross-section profile of the finished rolled steel strip. Its accuracy is no lower than that of known approaches, but the calculation complexity is significantly lower. The results can be used in the development of roll contouring for sheet rolling mills.

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