Imitating model of the rolling-drawing process for short sheets

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The paper is aimed at a detailed study of the rolling-drawing process for short sheets containing of different mono-metals. The examined process is characterized by absence of pulling forces, which are applied to ends of sheets, and by asymmetry, which is expresses in correlation of circumferential speed values of rolling rolls. Development of the imitation model of this process, allowing to determine its implementation variability in the conditions of rigidly preset kinematics, is considered as both the aim and scientific novelty of this research. The developed model also considers possibility of charging short mono-metallic sheets in a roll gap under an angle and with difference in friction coefficients during the contact with work rolls and allows to determine the contact stresses. Use of this imitation model makes it possible for researchers to determine beforehand workable and non-realistic variants for deformation of short mono-metallic sheets via asymmetric rolling-drawing process. To achieve the formulated aim, the article presents conclusion of relationships describing the components of power balance equation of the rolling-drawing process for short sheets in the conditions of absence of longitudinal stresses at the entrance and exit of deformation area. Building of a possible kinematic speed fields was also realized; it takes into account possibility of charging a billet in a roll gas under an angle to the rolling axis. The imitation model, which was obtained in this research, displayed that the feasible area of the examined process is a discrete one.
Key words: asymmetric rolling, rolling-drawing, power balance, boundary conditions, incompressibility conditions, metal plastic forming, possible kinematic speed field, deformation area.
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Introduction

Asymmetric rolling in the rolls rotating with various speed is considered as the one of prospective processes in the field of cold sheet rolling [1–6]. The rolling-drawing process is an ultimate variant of such process [7–10]; in this case the whole contact square with more fast-speed (driving) roll is a retarding area, while the whole contact square with less fast-speed roll (driven) is a forwarding area. Necessity of applying forward tension in an exit plane of deformation areas for standard process conditions is one of the problems connected with realization of the rolling-drawing process. It hinders applying this process for deformation of short sheets. In order to overcome the above-mentioned restriction, it is necessary for a driving roll to supply larger energy in deformation area in comparison with energy spent during the contact with a driven roll. It can be achieved in particular owing to increase of the friction coefficient during the contact with a driving roll.

The preliminarily conducted analysis of the equilibrium conditions in deformation area in the rolling-drawing process with various friction coefficients of the contact with work rolls allowed to reveal correlation between relationship of the contact friction coefficients and strip deformation degree for rigidly preset process kinematics (which correspond to rolling-drawing). Additionally, it is possible to determine the link between normal contact stresses for a driving and a driven rolls. In its turn, it is considered as the key indicator for managing the examined process. Determination of this link between stresses at the theoretical level is possible via creation of the imitating model of the rolling-drawing process for short sheets.

Thus, as soon as the literature survey displayed absence of such researches at present time, the main aim of this work is to form the imitating model of the rolling-drawing process for short sheets. The power balance equation can be used for this purpose.

Materials and methods

Determination of the power balance components suggests construction of possible kinematic speed fields.

At present time, various variants of building if such speed fields for asymmetric rolling [11]. However, they evidently don't take into account peculiarities of the rolling-drawing process, in particular rigidly preset slipping kinematics on the contact surfaces in deformation area. The calculating scheme (**Fig. 1**) is used as a base for building of possible kinematic speed fields.

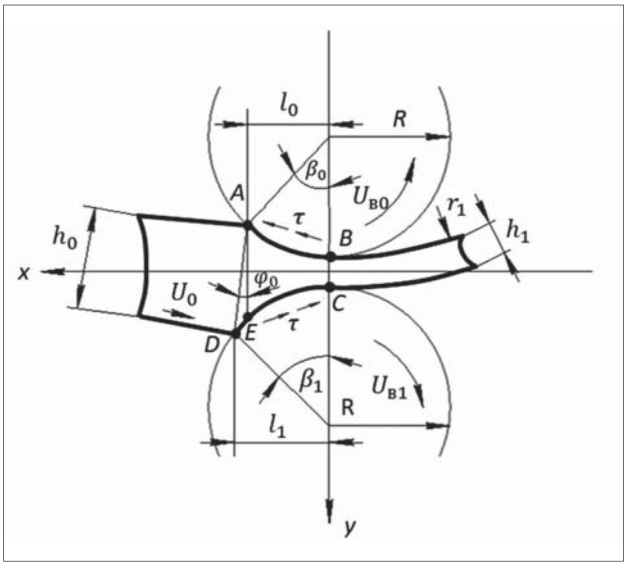

Fig 1. Calculation scheme of the deformation area

As soon as we can neglect by transversal metal flow during sheet rolling, incompressibility condition can be written as

$$\frac{\partial U_x}{\partial x} = \frac{-\partial U_y}{\partial y}$$

(U_x– projection of a speed vector on the direction $0x$; U_y – projection of a speed vector on the direction $0y$).

The following expressions can be taken as boundary conditions for rigidly preset kinematics in the deformation area, which corresponds to the rolling-drawing process:

1. In connection with the fact that the component of speed vector U_x at the exit of deformation area becomes constant, we can conclude:

$$\left.\frac{\partial U_x}{\partial x}\right|_{x=0} = 0 \quad .$$

2. Based on the fact that reduction by sheet thickness along the line BC is absence, we can conclude:

$$U_{y_{x=0}} = 0$$

3. A rear strip end moves with the constant speed U_0.
4. The speed vector in the point C is directed at a tangent to the roll surface and is equal to U_{b1}.

To build possible kinematic speed field, let us accept the following main hypotheses:

1. The component of speed vector U_x varies in direction x in correspondence with a squared relationship:

$$U_x = a_0 + a_1 x + a_2 x^2 \quad . \qquad (1)$$

2. The component of speed vector U_x varies in direction $0y$ axis at the exit of deformation area according to a linear relationship. In this case, in correspondence with the boundary condition 4 in the point C, we have $U_x = U_{b1}$. If we shall ensign the component of speed vector U_x in the point B as U_{xB}, then for x=0 we can write down:

$$U_{x_{x=0}} = \frac{1}{2}\left(U_{xB} + U_{b1}\right) - \left(U_{xB} - U_{b1}\right)\frac{y}{h_1}. \qquad (2)$$

In correspondence with the boundary condition 1 and taking into account the relationship (1), we can conclude:

$$\frac{\partial U_x}{\partial x_{x=0}} = 0 = a_1 + 2a_2 x \qquad (3)$$

then $a_1 = 0$ and the relationship (1) looks like:

$$U_x = a_0 + a_2 x^2 \; . \qquad (4)$$

3. In correspondence with the hypothesis 2 and the relationship (2), we shall have:

$$U_x = \frac{1}{2}\left(U_{xB} + U_{b1}\right) - \left(U_{xB} - U_{b1}\right)\frac{y}{h_1} + a_2 x^2 \; . \qquad (5)$$

For determination of the component of speed vector U_y, let us use incompressibility condition, according to which:

$$U_y = -2a_2 x \int dy + \Delta = -2a_2 xy + \Delta \; . \qquad (6)$$

The integration constant Δ is determined taking into account the fact that when x = 0, U_y = 0, in other words:

$$U_y = -2a_2 xy \; . \qquad (7)$$

The obtained possible kinematic speed field, which is described by the relationships (5) and (7), is built with accuracy to values a_2 and U_{xB}. For their determination, we shall use the boundary condition 4, taking into account that we can accept $\cos\beta_0 \approx 1$ owing to small size of the angle β_0. Thus we shall have:

$$a_2 = \frac{U_{b0} - \frac{1}{2}\left[U_{xB} + U_{b1} + \left(U_{xB} - U_{b1}\right)\left(1 + \frac{R}{h_1}\beta_0^2\right)\right]}{l_0^2} \; . \qquad (8)$$

To determine the value U_{xB}, we can also use the boundary condition 4. It should be mentioned that in the points A and D disruption of the speed field occurs. It is known [12], that a leap at disruption surface is possible only for a tangent of a speed vector component, while a normal component should remain constant. In this connection, let us consider metal flow kinematics in the point A (**Fig. 2**).

In correspondence with the Fig. 2 and the boundary condition 4, we can write down:

$$U_y = U_{b0} \sin\left(\beta_0\right)$$

Finally, due to small size of the angle β_0, we shall obtain:

$$a_2 = \frac{-U_{b0}\sqrt{\frac{\Delta h}{R}}}{\sqrt{\Delta h R} \cdot h_0} \; .$$

If we equalize the obtained and the previously found relationships for a_2, we shall get the required value U_{xB}:

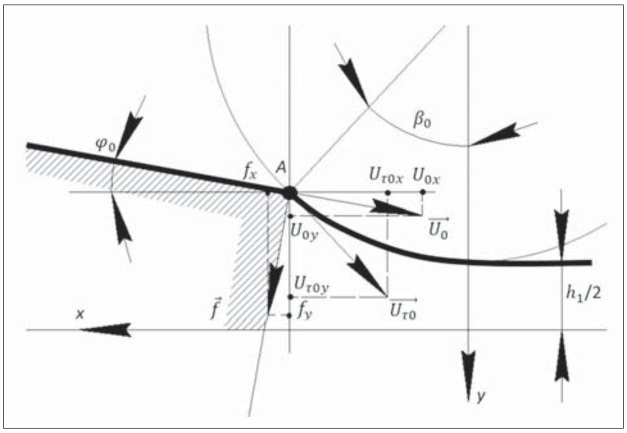

Fig. 2. Resolution of speed and stress vectors at deformation area entrance plane

$$U_{xB}=\frac{\dfrac{2\cdot l_0^2\cdot U_{b0}\cdot\sqrt{\dfrac{\Delta h}{R}}}{\sqrt{\Delta hR}\cdot h_0}+2\cdot U_{b0}+\dfrac{U_{b1}\cdot\Delta h}{h_1}}{2+\dfrac{\Delta h}{h_1}}\quad.\qquad(9)$$

Thus, possible kinematic speed field for the rolling-drawing process was built; it is described by the formulas (5) and (7), taking into account the formulas (8) and (9).

This built possible kinematic speed field allows to determine intensity of shift deformation speeds H, which is required for calculation of forming power:

$$H=2\sqrt{a_2^2\left(4x^2-y^2\right)+\frac{U_{b1}-U_{xB}}{2h_1}}\quad.$$

The equation of power balance contains the following components:

— forming power in the deformation area N_φ;
— shear power in the entrance plane N_o;
— shear power in the exit plane N_1;
— power of friction forces during the contact with a driven roll $N_{\tau o}$;
— power of friction forces during the contact with a driving roll $N_{\tau 1}$.

In this case we shall accept that consumed power will be negative, while supplied power will be positive.

It is known [12] that forming power in general is determined by the following expression:

$$N_\varphi=\int_V\tau_s H dV$$

Taking into account the previously obtained expression for H, the forming power will be equal to:

$$N_\varphi=2\int_V\tau_s\sqrt{a_2^2\left(4x^2-y^2\right)+\frac{U_{b1}-U_{xB}}{2h_1}}dV\quad.$$

The integral by volume in this expression can be calculated using numerical methods, i.e. taking into account metal strengthening. At the same time, an approximate analytical relationship can be obtained. When solving similar tasks, it is possible usually to neglect shear deformations and to average metal resistance during shift plastic deformation by the volume of deformation area, when calculating an integral value [13]. However, as soon as increased shift deformations are considered as significant features of the rolling-drawing process, it is incorrectly to exclude them completely from consideration. Thus, to take into account the effect of these deformations, the constant coefficient ζ, is introduced, which is calculated via the obtained speed field:

$$\zeta=\frac{\dfrac{\dfrac{1}{2}\cdot U_{21-U_{xB}}}{a_2}\cdot 1+\dfrac{\dfrac{2}{3}\cdot l^2}{Rh_1}}{h_1+\dfrac{l^2}{R}}$$

As a result, the final expression for determination of a forming power looks like:

$$N_\varphi=\zeta\tau_{sc}ba_2l_0^2\left(h_1+4\frac{l_0^2}{R}\right)\quad.\qquad(10)$$

Let us determine the power N_0, which was consumed in the entrance plane of the deformation area for varying the speed direction vector. It is known [12], that the power of surface forces is determined in general by the expression:

$$N=\int_S\left(\vec{f},\Delta\vec{U}\right)dS$$

In the examined case, as soon as extending or compressing forces are not applied to the rear strip end, the scalar product in the element of integration looks like (see the Fig. 2):

$$\left(\vec{f},\Delta\vec{U}\right)=f_x\Delta U_x+f_y\Delta U_y\quad.$$

Respectively, we can write down:

$$f_x=\tau_{s0}\sin\varphi_0\;;$$

$$f_y=\tau_{s0}\cos\varphi_0\;;$$

$$\Delta U_x=U_{b0}\cos\varphi_0-\frac{1}{2}\left(U_{xB}+U_{b1}\right)+\left(U_{xB}-U_{b1}\right)\frac{y}{h_1}-a_2x^2$$

$$\Delta U_y=U_{b0}\sin\varphi_0+2a_2xy$$

Thus, the power N_0 which was consumed in the entrance plane, will be equal to:

$$N_0=\int_{S_0}\left(f_x\Delta U_x+f_y\Delta U_y\right)dS$$

It should be taken into account in this case, that the integral by square S_0 covers square, half of which is restricted by the contour of AD line and another half includes width of upper and lower strip surfaces. According to the rule of integral calculation by contour, and taking into account that $y_A=\frac{-1}{2}h_0\cos\varphi_0$ and $y_D=\frac{h_0}{2\cos\varphi_0}$, it is possible to write down:

$$\begin{aligned}N_0={}&\frac{1}{2}\tau_{s0}bh_0\left(1+\cos^2\varphi_0\right)\Bigg\{\left[U_{b0}\cos\varphi_0-\frac{1}{2}\left(U_{xB}+U_{b1}\right)\right]\left(l_0-l_1\right)+\\&+\frac{\dfrac{\dfrac{1+\cos^2\varphi_0}{2\cos\varphi_0}\cdot U_{xB}-U_{b1}}{4\left(l_0-l_1\right)}\cdot h_0}{h_1}\left(l_0^2-l_1^2\right)\sin^2\varphi_0-\frac{1+\cos^2\varphi_0}{\cos\varphi_0}l_1+\left(l_0-l_1\right)\cos\varphi_0+\\&+\frac{a_2}{3}\left(l_0^3-l_1^3\right)\sin^2 2\varphi_0+U_{b0}\left(l_0-l_1\right)\sin\varphi_0+\\&+a_2h_0\left[\frac{\dfrac{1+\cos^2\varphi_0}{3\cos\varphi_0}\cdot l_0^3-l_1^3}{l_0-l_1}-\frac{1}{2}\left(\frac{\dfrac{1+\cos^2\varphi_0}{\cos\varphi_0}\cdot l_1}{l_0-l_1}-\cos\varphi_0\right)\left(l_0^2-l_1^2\right)\right]\Bigg\}\end{aligned}\qquad(11)$$

The useful power of friction forces on the contact surface with a driving roll is calculated as a power of friction forces with metal speed, i.e.:

$$N_{\tau 1} = \int_{S_{\tau 1}} \left(\vec{f}, \vec{U}\right) dS = b\int_0^{l_1} \left(f_x U_x + f_y U_y\right) dS \quad .$$

To determine projections of a surface stress vector on coordinate directions, which were preset by a unit basis vectors $\vec{e}_x$ and $\vec{e}_y$ let us consider the conditions on a sloping bench, which corresponds to the point A in the **Fig. 3**.

$$\begin{cases} f_x = f_n \cos\left(\overrightarrow{e_x}, \overrightarrow{e_n}\right) + f_\tau \cos\left(\overrightarrow{e_x}, \overrightarrow{e_\tau}\right); \\ f_y = f_n \cos\left(\overrightarrow{e_y}, \overrightarrow{e_n}\right) + f_\tau \cos\left(\overrightarrow{e_y}, \overrightarrow{e_\tau}\right), \end{cases}$$

As a result, we shall obtain:

$$N_{\tau 1} = p_1 l_1 b \{ \frac{1}{2} \left(\sin\beta_1 - f_1 \cos\beta_1\right)$$
$$\left[U_{xB} + U_{b1} - \frac{U_{xB} - U_{b1}}{h_1} \left(h_1 + \frac{l_1^2}{3R} \right) + \frac{2}{3} a_2 l_1^2 \right] -$$
$$- \frac{1}{2} a_2 \left(\cos\beta_1 + f_1 \sin\beta_1\right) \left(h_1 l_1 + \frac{l_1^3}{2R} \right) \}. \quad (12)$$

In the same way we can write down the expression for reactive power of friction forces on the contact surface with a driven roll:

$$N_{\tau 0} = \frac{1}{2} p_0 l_0 b \left\{ \left(\sin\beta_0 + f_0 \cos\beta_0\right) \left[2U_{xB} + \right.\right.$$
$$\left.\left. + \frac{l_0^2}{3} \left(\frac{U_{xB} - U_{b1}}{h_1 R} + a_2 \right) \right] + a_2 l_0 \left(\cos\beta_0 - f_0 \sin\beta_0\right) \right.$$
$$\left. \left(h_1 + \frac{l_0^2}{2R} \right) \right\}. \quad (13)$$

According to the power balance equation, the sum of (10)–(13) expression should be equal to zero. Thus, after this summarizing we shall obtain the equation containing one unknown quantity p_0 (we accept that p_0 and p_1 relation is determined beforehand).

As a result, we get the imitating model of the rolling-drawing process for short sheets, allowing to determine contact stresses in the deformation area and reduction value for preset correlation of friction coefficients on the opposite contact surfaces.

In order to evaluate and check workability of the obtained imitating model, rolling forces for a driving rolling roll were calculated via the concluded formulas from the previous research [14]. Then the range including the most realistic values of a rolling force (P_1 = 255 ÷ 610 MPa) was formed and the workable variants, when deformation of short sheets will be implemented via asymmetric rolling-drawing process, was determined on this base.

When calculating, the values of rolling roll diameters within the range 100–900 mm (with 50 mm step) were varied, as well as relationships of the friction coefficients within the range 1.25–2.5 (with step 0.25). Steel 20 grade sheets with thickness 4.0 mm and width 800 mm were used.

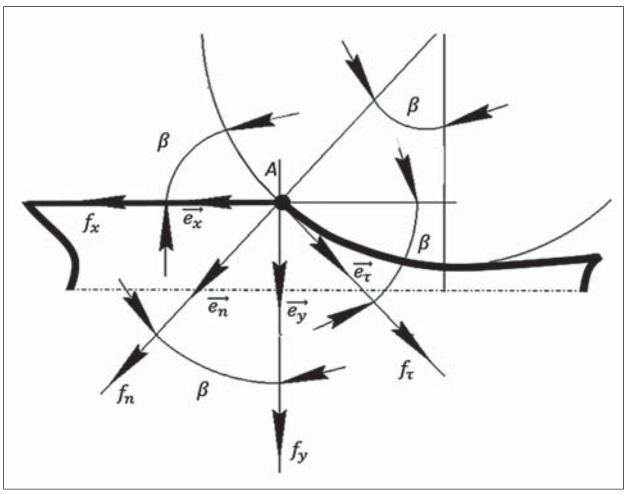

Fig. 3. Decomposition of the surface stress vector in coordinate directions on a sloping bench

Results and discussion

It was revealed on the base of conducted calculations, that the feasible area of the rolling-drawing process is rather narrow and discrete in the conditions of absence of longitudinal stresses on the boundaries of deformation area (**Table**). This results is caused by rigid process kinematics and presence of only one power supply source.

Realized variants of the rolling-drawing process

Rolling roll diameter *D*, mm	Correlation between friction coefficients for the driving (f_1) and driven (f_0) rolling rolls (f_1/f_0)	Relative deformation Ɛ, %
500	1.79	21.29
550	1.88	29.26
600	1.92	34.87
650	1.97	42.24

It was noted that increase of roll diameter leads to shifting of the feasibility area to the zone of increased reduction values (up to 42 %). Respectively, correlation of friction coefficients also enlarges. The noted regularity is explained by the fact, that increase of a driving roll diameter leads to increase of a contact surface length and, respectively, to rise of power supply in the deformation area. However, to prevent substantial enlargement of consumed power of slipping friction from the side of a driven roll, it is required to increase additionally a difference between friction coefficients in the contact with a driving roll and a driven roll respectively.

Additional analysis of non-realizing deformation variants for short mono-metallic sheets via the rolling-drawing process helped to reveal that absence of a suitable correlation between friction coefficients at the contact surfaces for each not presented *D* value is considered as a cause of their non-feasibility (see the Table). The list of accessible friction coefficients was presented in the previous research [14].

The calculations that were carried out for assessment of a rolling force for the examined cases, have expectedly displayed that the level of an average contact pressure is situated at the level of metal resistance to plastic deformation, and it is essentially lower than an average contact pressure for symmetric sheet rolling with the same deformation conditions.

Conclusions

Thus, the imitating model for asymmetric rolling-drawing process of short mono-metallic sheets at the rigid preset kinematic conditions was developed, and it allowed to testify the following results.

1. The area of realization of this process is narrow and discrete, thereby it has limited number of workable variants.

2. Increase of rolling roll diameters leads to increase of reduction of short sheets and difference between friction coefficients at the contact surfaces.

3. The average contact pressure values during asymmetric rolling-drawing process are substantially lower than the average contact pressure values during symmetric rolling.

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