Analytical dependence of relative elongations on normal stresses at elastic-plastic stretching of bimetallic beam with transverse connection of metals

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Bimetals are composite materials consisting of two or more layers of various metals or alloys connected in an integral way (by welding, rolling, soldering, and so on). Their key advantages are due to the combination of properties of the incoming components. To list the main advantages of bimetals. Optimization of operational properties – the combination of dissimilar materials allows you to "construct" a material with specified characteristics: high strength of the inner layer (for example, steel) and excellent thermal conductivity of the outer layer (for example, aluminium); corrosion-resistant cladding and a durable base; wear resistance of the surface; damping properties of the base. Cost-effectiveness – replacing monolithic expensive material with bimetal (for example, stainless steel and carbon steel) reduces cost while maintaining key properties; saving alloying elements (valuable alloy is applied only to the work surface). Increased corrosion resistance – a protective layer of corrosion-resistant metal (stainless steel, titanium, nickel) protects the base from aggressive media, increasing the service life of the product. Improved thermal conductivity – in radiators and heat exchangers, the aluminium or copper outer layer ensures rapid heat dissipation, and the steel inner channel withstands high pressure and corrosion. High strength and wear resistance – the base made of high-strength metal (steel, titanium) can withstand mechanical loads, and the surface layer can be optimized for friction, erosion or abrasive wear. Thermal stability – the combination of materials with different coefficients of thermal expansion makes it possible to create elements resistant to thermal deformations (for example, in thermal compensators). Weight reduction of the structure – the use of light metals (aluminium, magnesium) in the outer layers while maintaining strength due to the steel or titanium base. Processability of processing – the possibility of stamping, bending, welding of bimetallic sheets without loss of joint properties; convenience of mechanical processing (the hard layer is cut off, the soft one is deformed). Aesthetics and protection decorative coating (chrome plating, nickel plating) on a solid base improves the appearance and protects against scratches. Expansion of functionality – in electrical engineering: copper and aluminium to reduce resistance while saving copper; in the tool industry: high-speed steel and structural steel to save expensive alloy; in shipbuilding: corrosion-resistant layer and high-strength base. To list are some examples of the use of bimetals. Heating radiators – the steel provides increased strength and corrosion resistance, and the aluminum provides increased heat transfer. Pipelines carbon steel provides increased load-bearing capacity, and stainless steel provides corrosion protection. Knives and cutting tools solid cutting edge and viscous base. Electrical contacts – copper provides increased conductivity, and tungsten provides increased wear resistance. Thus, bimetals allow combining the best properties of different materials, achieving an optimal balance of strength, corrosion resistance, thermal conductivity and cost. This makes them indispensable in metallurgy, mechanical engineering, energy and other industries. In this paper, the analytical dependence of relative elongations on normal stresses at stretching of a bimetallic beam with a transverse connection of metals is obtained.

Key words: bimetallic beam, transverse connection of metals, elastic-plastic stretching, normal stresses, relative elongations, forward and reverse approximations under stretching.

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Introduction

The various methods are used to connect metals in a bimetallic beam, which make it possible to obtain a strong and reliable connection of dissimilar materials. The choice of the joining method depends on the specific requirements for the bimetallic beam, the properties of the metals being joined, the operating conditions, the design requirements and the technological capabilities of the production. Welding is one of the main methods of joining metals in bimetals. At the same time, it is important to minimize the formation of intermetallic phases, reduce the time of exposure to high temperatures on the weld and avoid direct mixing of metals.

Argon arc welding is used to work with thin parts (from 0.5 mm) and is performed in any spatial position.

Laser welding provides a point concentration of temperature, which minimizes the thermal effect on the bimetallic assembly. When joined together, the laser beam is directed at an alloy with a lower melting point.

Friction welding is used for cylindrical parts and connects metals in a solid phase without melting. One of the workpieces remains motionless, the second is rotated and pressed

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against the first. When the movement speed of the part is about 1,500 revolutions per minute, the melting of the workpieces begins. To improve the strength characteristics of the seam and reduce the formation of a brittle intermetallic joint, intermediate inserts (made of the same metals as the joint itself or alloys with intermediate properties) or surfacing for the transition layer, as well as controlled cooling can be used.

Explosion welding is a method based on the use of explosion energy. When a moving part collides with a stationary one, kinetic energy is generated, which goes to plastic deformation of the metal layers being joined, which leads to welding. Explosion welding allows you to connect metals with very different properties without heating and deformation; provides the connection of almost any metals and alloys, including difficult-to-weld pairs (for example, aluminium and steel); forms a joint in a very short time (fractions of a second). However, explosion welding can lead to the hard and brittle intermetallic compound, which negatively influence on the quality of the connection. After explosion welding, the bimetallic compound is sometimes annealed to eliminate residual stresses, obtain a uniform structure and increase strength characteristics.

Diffusion welding – the process takes place at high temperatures and under pressure for a long time. This ensures the diffusion of atoms of one metal into another, creating a strong compound at the molecular level. Features of diffuse welding – the welding temperature reaches 0.7–0.8 melting point of the more fusible structural material; during operation, tensile or compressive stresses may occur, which affect the operational characteristics of the joint; intermediate layers are sometimes used to prevent the formation of intermetallides. For example, a porous ribbon made of ultrafine metal powder.

Pressure rolling (batch rolling) – the two metals are rolled together at high pressure to create a single sheet. The temperature and pressure are selected individually for each pair of metals to ensure their maximum adhesion.

Mechanical connections – in some cases, it is advisable to use mechanical connections (bolted, riveted, and others). They provide the ability to disassemble the structure, which is critical for maintenance and repair of equipment. When two different metals come into contact in a mechanical joint, it is important to avoid the formation of galvanic corrosion. To do this, a polymer coating is applied to the parts.

Adhesive bonding – glue can be used to joint metals if high adhesion and sufficient surface area are required to form a strong joint. However, the adhesive has a limited operating temperature (no higher than +120 °C), therefore, under high thermal stress, a welded or mechanical joint should be chosen for the bimetallic assembly.

The purpose of this work is to construct the analytical inverse dependence of relative elongations on normal stresses at stretching of a bimetallic beam with a transverse connection of metals.

Mechanical characteristics of metals

For the calculation of the relative elongations ε of the bimetallic beam with a cross connection of metals under static stretching by the force F (the normal stress $\sigma = F/S_0$, where S_0 – the cross-sectional area of the beam before stretching) need to know the inverse dependence of the relative elongations on the normal stresses $\varepsilon = \varepsilon(\sigma)$ [1–5].

Below are four analytical dependences of the normal stresses $\sigma = \sigma(\varepsilon)$ and four analytical dependences of the relative elongations $\varepsilon = \varepsilon(\sigma)$, depending on the Young's modulus E, the yield strength σ_y, the relative elongation $\varepsilon_y = \sigma_y / E$ in the yield strength, the hardening module P_y in the yield strength, the ultimate strength σ_u, and the relative elongation ε_u in the ultimate strength.

In the direct approximations of the steel hardening zone $\sigma = \sigma(\varepsilon)$, the "reference" point is the relative elongations ε_y corresponding to the yield strength σ_y. The deviation of relative elongations in the steel hardening zone is measured relative to ε_y, the deviation of normal stresses is measured relative to σ_y.

In the case of the inverse approximations of the steel hardening zone $\sigma = \sigma(\varepsilon)$, the "reference" point is the relative elongations ε_u corresponding to the ultimate strength σ_u. The deviation of relative elongations in the steel hardening zone is measured relative to ε_u, the deviation of normal stresses is measured relative to σ_u.

Three boundary conditions are used in the analytical description of the hardening zone. The first two boundary conditions are the same in all variants – the hardening curve $\sigma = \sigma(\varepsilon)$ passes through the points corresponding to the yield strength and the ultimate strength. The third boundary condition is variable – either the hardening modulus is set at the point corresponding to the yield strength, or the hardening curve $\sigma = \sigma(\varepsilon)$ has a maximum at the point corresponding to the ultimate strength.

The first variation of the direct quadratic approximation

At elastoplastic bending, the direct dependence of normal stresses on relative elongations $\sigma = \sigma(\varepsilon)$ has the form:

$$\sigma(\varepsilon) = \begin{cases} E\varepsilon, \quad 0 \le \varepsilon < \varepsilon_y = \dfrac{\sigma_y}{E}; \\ \sigma_y + 2\dfrac{\sigma_u - \sigma_y}{(\varepsilon_u - \varepsilon_y)}(\varepsilon - \varepsilon_y) - \dfrac{\sigma_u - \sigma_y}{(\varepsilon_u - \varepsilon_y)^2}(\varepsilon - \varepsilon_y)^2, \end{cases} \tag{1}$$

$$\varepsilon_u \ge \varepsilon \ge \varepsilon_y; \quad \sigma(\varepsilon_y) = \sigma_y, \quad \sigma(\varepsilon_u) = \sigma_u, \quad \frac{d\sigma}{d\varepsilon}(\varepsilon_u) = 0.$$

The inverse dependence of relative elongations on normal stresses $\varepsilon = \varepsilon(\sigma)$ has the form:

$$\varepsilon(\sigma) = \begin{cases} \dfrac{\sigma}{E}, \quad 0 \le \sigma < \sigma_y; \\ \varepsilon_y + \left(1 - \sqrt{\dfrac{\sigma_u - \sigma}{\sigma_u - \sigma_y}}\right)(\varepsilon_u - \varepsilon_y), \quad \sigma_u \ge \sigma \ge \sigma_y; \end{cases} \tag{2}$$

$$\varepsilon(\sigma_y) = \varepsilon_y, \quad \varepsilon(\sigma_u) = \varepsilon_u, \quad \frac{d\varepsilon}{d\sigma}(\sigma_u) = +\infty.$$

The second variation of the direct quadratic approximation

At elastoplastic bending, the direct dependence of normal stresses on relative elongations $\sigma = \sigma(\varepsilon)$ has the form:

$$\sigma(\varepsilon)=\begin{cases} E\varepsilon, & 0\le\varepsilon<\varepsilon_y=\dfrac{\sigma_y}{E};\\ \sigma_y+P_y\left(\varepsilon-\varepsilon_y\right)-\dfrac{P_y\left(\varepsilon_u-\varepsilon_y\right)-\left(\sigma_u-\sigma_y\right)}{\left(\varepsilon_u-\varepsilon_y\right)^2}\left(\varepsilon-\varepsilon_y\right)^2, & \varepsilon_u\ge\varepsilon\ge\varepsilon_y;\end{cases}$$

$$\sigma\left(\varepsilon_y\right)=\sigma_y,\quad \sigma\left(\varepsilon_u\right)=\sigma_u,\quad \frac{d\sigma}{d\varepsilon}\left(\varepsilon_y\right)=P_y. \qquad (3)$$

The inverse dependence of relative elongations on normal stresses $\varepsilon=\varepsilon(\sigma)$ has the form:

$$\varepsilon(\sigma)=\begin{cases} \dfrac{\sigma}{E}, & 0\le\sigma<\sigma_y;\\ \varepsilon_y+\dfrac{\left(\varepsilon_u-\varepsilon_y\right)^2}{2\left(P_y\left(\varepsilon_u-\varepsilon_y\right)-\left(\sigma_u-\sigma_y\right)\right)}\left(P_y-\sqrt{P_y^2-4\dfrac{\left(P_y\left(\varepsilon_u-\varepsilon_y\right)-\left(\sigma_u-\sigma_y\right)\right)\left(\sigma-\sigma_y\right)}{\left(\varepsilon_u-\varepsilon_y\right)^2}}\right), & \sigma_u\ge\sigma\ge\sigma_y;\end{cases} \qquad (4)$$

$$\varepsilon(\sigma_y)=\varepsilon_y,\quad \frac{d\varepsilon}{d\sigma}(\sigma_y)=\frac{1}{P_y}.$$

If $P_y(\varepsilon_u-\varepsilon_y)-2(\sigma_u-\sigma_y)\le 0$, then $\varepsilon(\sigma_u)=\varepsilon_u$.

The first variation of the inverse quadratic approximation

At elastoplastic bending, the direct dependence of normal stresses on elongations $\sigma=\sigma(\varepsilon)$ has the form:

$$\sigma(\varepsilon)=\begin{cases} E\varepsilon, & 0\le\varepsilon\le\varepsilon_y=\dfrac{\sigma_y}{E};\\ \sigma_u-\dfrac{\sigma_u-\sigma_y}{(\varepsilon_u-\varepsilon_y)^2}(\varepsilon_u-\varepsilon)^2, & \varepsilon_u\ge\varepsilon\ge\varepsilon_y;\end{cases} \qquad (5)$$

$$\sigma(\varepsilon_y)=\sigma_y,\quad \sigma(\varepsilon_u)=\sigma_u,\quad \frac{d\sigma}{d\varepsilon}(\varepsilon_u)=0.$$

The inverse dependence of relative elongations on normal stresses $\varepsilon=\varepsilon(\sigma)$ has the form:

$$\varepsilon(\sigma)=\begin{cases} \dfrac{\sigma}{E}, & 0\le\sigma\le\sigma_y;\\ \varepsilon_u-\sqrt{\dfrac{\sigma_u-\sigma}{\sigma_u-\sigma_y}}\left(\varepsilon_u-\varepsilon_y\right), & \sigma_u\ge\sigma\ge\sigma_y;\end{cases} \qquad (6)$$

$$\varepsilon(\sigma_y)=\varepsilon_y,\quad \varepsilon(\sigma_u)=\varepsilon_u,\quad \frac{d\varepsilon}{d\sigma}(\sigma_u)=+\infty.$$

The second variation of the inverse quadratic approximation

At elastoplastic bending, the direct dependence of normal stresses on relative elongations $\sigma=\sigma(\varepsilon)$ has the form:

$$\sigma(\varepsilon)=\begin{cases} E\varepsilon, & 0\le\varepsilon<\varepsilon_y=\dfrac{\sigma_y}{E};\\ \sigma_u+\dfrac{P_y\left(\varepsilon_u-\varepsilon_y\right)-2\left(\sigma_u-\sigma_y\right)}{\left(\varepsilon_u-\varepsilon_y\right)}\left(\varepsilon_u-\varepsilon\right)-\dfrac{P_y\left(\varepsilon_u-\varepsilon_y\right)-\left(\sigma_u-\sigma_y\right)}{\left(\varepsilon_u-\varepsilon_y\right)^2}\left(\varepsilon_u-\varepsilon\right)^2, & \varepsilon_u\ge\varepsilon\ge\varepsilon_y;\end{cases} \qquad (7)$$

$$\sigma(\varepsilon_y)=\sigma_y,\quad \sigma(\varepsilon_u)=\sigma_u,\quad \frac{d\sigma}{d\varepsilon}(\varepsilon_u)=P_y.$$

$$\varepsilon(\sigma)=\begin{cases} \dfrac{\sigma}{E}, & 0\le\sigma<\sigma_y;\\ \varepsilon_u-\dfrac{\left[\left(P_y\left(\varepsilon_u-\varepsilon_y\right)-2\left(\sigma_u-\sigma_y\right)\right)+\sqrt{\left(P_y\left(\varepsilon_u-\varepsilon_y\right)-2\left(\sigma_u-\sigma_y\right)\right)^2+4\left(P_y\left(\varepsilon_u-\varepsilon_y\right)-\left(\sigma_u-\sigma_y\right)\right)\left(\sigma_u-\sigma\right)}\right]\left(\varepsilon_u-\varepsilon_y\right)}{2\left(P_y\left(\varepsilon_u-\varepsilon_y\right)-\left(\sigma_u-\sigma_y\right)\right)}, & \sigma_u\ge\sigma\ge\sigma_y;\end{cases} \qquad (8)$$

$$\varepsilon(\sigma_y)=\varepsilon_y,\quad \frac{d\varepsilon}{d\sigma}(\sigma_y)=\frac{1}{P_y}.$$

The inverse dependence of relative elongations on normal stresses $\varepsilon=\varepsilon(\sigma)$ has the form:

If $P_y(\varepsilon_u-\varepsilon_y)-2(\sigma_u-\sigma_y)\le 0$, then $\varepsilon(\sigma_u)=\varepsilon_u$.

Stretching of bimetallic beam with a transverse connection of metals

Let's consider the geometric characteristics of a bimetallic beam with a transverse connection of metals (**Fig. 1, Fig. 2**). Let the initial length of the beam be l_0, the initial length of the first (left) part of the beam be l_{c0}, and the initial length of the second (right) part of the beam be l_{s0}. Then $l_0=l_{c0}+l_{s0}$. Let the initial cross-sectional area of the beam (the first and second parts of the beam) S_0 be the same along the entire length of the beam.

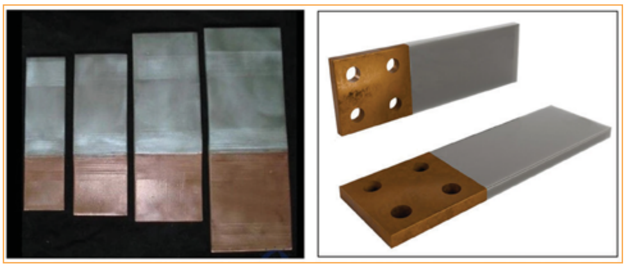

Fig. 1. Bimetallic beam with transverse connection of metals

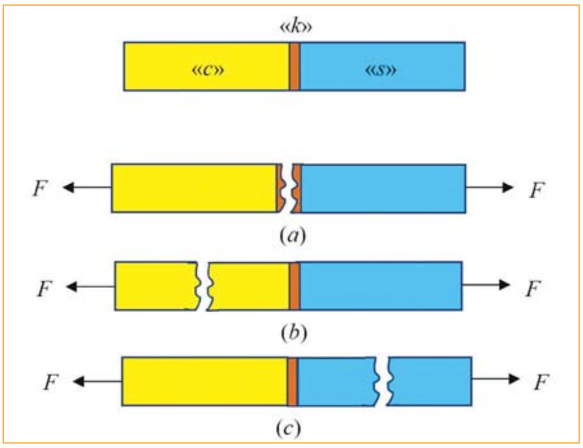

Fig. 2. Schemes of destruction of bimetallic beam with transverse connection of metals under stretching: F – stretching force, "c" and "s" – left and right metal components of beam, "k" – thin transverse connection of beam's components

Let's consider the mechanical characteristics of the metal of the first and second parts of the beam. Let E_c, σ_{yc}, σ_{uc}, $\varepsilon_{yc}=\sigma_{yc}/E_c$, ε_{uc}, P_{yc} be the Young's modulus, the yield strength, the ultimate strength, the relative elongation at the yield strength, the relative elongation at the ultimate strength and the hardening modulus at the yield strength for metal from the left part of the bimetal beam [1, 6–11]. Let E_s, σ_{ys}, σ_{us}, $\varepsilon_{ys} = \sigma_{ys} / E_s$, ε_{us}, P_{ys} be the Young's modulus, the yield strength, the ultimate strength, the relative elongation at the yield strength, the relative elongation at the ultimate strength and the hardening modulus at the yield strength for metal from the right part of the bimetal beam.

Let stretch the bimetallic beam longitudinally by the force F. Then the normal stress of the cross-section of the bimetallic beam along its entire length (in the first and second parts of the beam) is constant and equal to $\sigma = F / S_0$ [1, 8, 12, 13].

Let Δl_c and Δl_s be the absolute elongations of the left and right parts of the bimetallic beam under stretching, and Δl be the absolute elongation of the entire beam under stretching ($\Delta l = \Delta l_c + \Delta l_s$). Then the relative elongations of the left and right sides of the bimetallic beam and the relative elongation of the entire beam are equal to

$$\xi_{c0} = \frac{l_{c0}}{l_{c0} + l_{s0}}, \quad \xi_{s0} = \frac{l_{s0}}{l_{c0} + l_{s0}}, \quad \xi_{c0} + \xi_{s0} = 1, \tag{9}$$

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{\varepsilon_c l_{c0} + \varepsilon_s l_{s0}}{l_{c0} + l_{s0}} = \varepsilon_c \xi_{c0} + \varepsilon_s \xi_{s0}, \tag{10}$$

The Young's modulus of the bimetallic beam is

$$E = \frac{E_c E_s \left(l_{c0} + l_{s0}\right)}{E_c l_{s0} + E_s l_{c0}} = \frac{E_c E_s}{E_c \xi_{s0} + E_s \xi_{c0}}. \tag{11}$$

Let σ_{uk} be the ultimate strength of the connection's material of two metals of the bimetallic beam at their junction.

If $\sigma_{uk} < \min(\sigma_{uc}, \sigma_{us})$, then under stretching of the bimetallic beam, the bimetallic beam will collapse at the junction of two different metals in the connection's material with the index "k" (Fig. 2, *a*) [1, 14–17].

If $\sigma_{uc} < \min(\sigma_{uk}, \sigma_{us})$, then under stretching of the bimetallic beam, the bimetallic beam will collapse in the metal with the index "c" (Fig. 2, *b*).

If $\sigma_{us} < \min(\sigma_{uk}, \sigma_{uc})$, then under stretching of the bimetallic beam, the bimetallic beam will collapse in the metal with the index "s" (Fig. 2, *c*).

The yield strength σ_y, the ultimate strength σ_u, the relative elongation ε_y at the yield strength and the relative elongation ε_u at the ultimate strength of the bimetallic beam are equal:

$$\sigma_y = \min\left(\sigma_{yc}, \sigma_{ys}\right), \quad \sigma_u = \min\left(\sigma_{uc}, \sigma_{us}\right), \tag{12}$$

$$\varepsilon_y = \min\left(\varepsilon_{yc}, \varepsilon_{ys}\right) = \min\left(\frac{\sigma_{yc}}{E_c}, \frac{\sigma_{ys}}{E_c}\right), \quad \varepsilon_u = \min\left(\varepsilon_{uc}, \varepsilon_{us}\right). \tag{13}$$

The hardening modulus P_y at the yield strength of the bimetallic beam is equal to

$$P_y = \begin{cases} \dfrac{P_{yc} E_s}{P_{yc} \xi_{s0} + E_s \xi_{c0}}, & \varepsilon_y = \varepsilon_{yc} < \varepsilon_{ys}; \\ \dfrac{E_c P_{ys}}{E_c \xi_{s0} + P_{ys} \xi_{c0}}, & \varepsilon_y = \varepsilon_{ys} < \varepsilon_{yc}; \\ \dfrac{P_{yc} P_{ys}}{P_{yc} \xi_{s0} + P_{ys} \xi_{c0}}, & \varepsilon_y = \varepsilon_{ys} = \varepsilon_{yc}. \end{cases} \tag{14}$$

The dependence of the relative elongations of the bimetallic beam on normal stresses (the reverse diagram "relative elongations – normal stresses") has the form

$$\varepsilon(\sigma) = \varepsilon_c(\sigma)\xi_{c0} + \varepsilon_s(\sigma)\xi_{s0}, \tag{15}$$

where $\varepsilon_c(\sigma)$ and $\varepsilon_s(\sigma)$ are the inverse dependences of relative elongations on normal stresses for left and right parts of the bimetallic beam, determined by the formulas (2), (4), (6) and (8).

Knowing the analytical dependence $\varepsilon(\sigma)$ of the relative elongations on the normal stresses for the bimetallic beam, it is possible to numerically construct the dependence $\sigma(\varepsilon)$ of the normal stresses on the relative elongations for the bimetallic beam.

Numerical calculations

Under stretching a metal beam, the value δ (the elongation at rupture) is defined as the ratio of the length of the beam at break to the original length of the beam. The value ψ (a relative narrowing at rupture) is defined as the ratio of the difference between the initial cross-sectional area of the beam and the minimum cross-sectional area of the beam in the neck area at rupture to the initial cross-sectional area of the beam [1, 2, 6]. The values of δ and ψ characterize the plasticity of steel [1, 16, 17].

Let's consider a bimetallic beam with a transverse connection of tubular steel of strength class K60 and titanium alloy BT6L (Ti-6Al-4V). Then for the steel K60 (index "s") $E_s = 2.10 \cdot 10^{11}$ Pa, $\sigma_{ys} = 500$ MPa, $\sigma_{us} = 600$ MPa, $\delta_s = 0.35$, $\psi_s = 0.40$, $\varepsilon_{ys} = \sigma_{ys} / E_s = 0.00238$, $\varepsilon_{us} \approx \delta_s(1 - \psi_s) = 0.210$, $P_{ys} \approx 2(\sigma_{us} - \sigma_{ys})/(\varepsilon_{us} - \varepsilon_{ys}) = 0.963 \cdot 10^9$ Pa; and for the titanium alloy (index "c") $E_c = 2.17 \cdot 10^{11}$ Pa, $\sigma_{yc} = 828$ MPa, $\sigma_{uc} = 1138$ MPa, $\delta_c = 0.15$, $\psi_c = 0.45$, $\varepsilon_{yc} = \sigma_{yc} / Ec = 0.00382$, $\varepsilon_{uc} \approx \delta_c(1 - \psi_c) = 0.0825$, $P_{ys} \approx 2(\sigma_{uc} - \sigma_{yc})/(\varepsilon_{uc} - \varepsilon_y c) = 7.880 \cdot 10^9$ Pa.

Let the length of the steel section be half the total length of the bimetallic beam ($\xi_s = \xi_c = 1/2$). We will use the first variation of the inverse quadratic approximation to describe the dependence of normal stresses on relative elongations for steel and titanium alloy. The dependencies $\sigma_c(\varepsilon)$ and $\sigma_s(\varepsilon)$ are shown in **Fig. 3**.

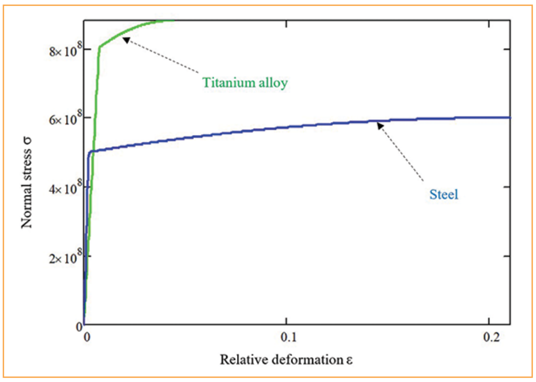

Fig. 3. Analytical dependence of normal stresses on relative deformations for beam made of tubular steel of strength class K60 and for beam made of titanium alloy Ti–6Al–4V

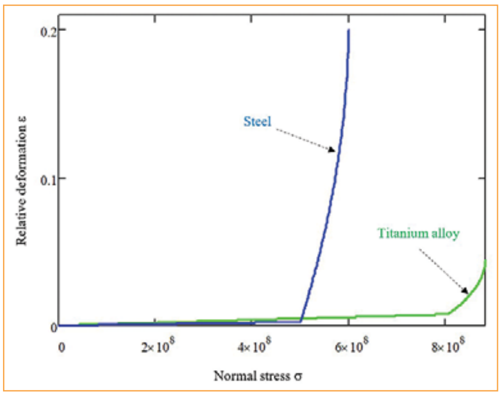

Fig. 4. Analytical dependence of relative deformations on normal stresses for beam made of steel of strength class K60 and for beam made of titanium alloy Ti-6Al-4V

We will use the first variation of the inverse quadratic approximation to describe the dependence of relative elongations on normal stresses for steel and titanium alloy. The dependencies $\varepsilon_c(\sigma)$ and $\varepsilon_s(\sigma)$ are shown in **Fig. 4**.

The inverse dependence $\varepsilon(\sigma)$ of relative elongations on normal stresses for the bimetallic beam, made from steel and titanium alloy, is shown in **Fig. 5**. The direct dependence $\sigma(\varepsilon)$ of normal stresses on relative elongations for the bimetallic beam, made of steel and titanium alloy, is shown in **Fig. 6**.

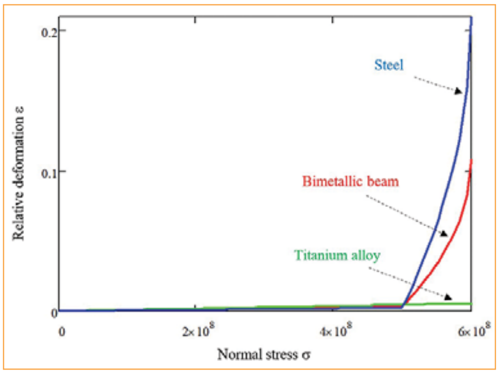

Fig. 5. Analytical dependence of relative deformations on normal stresses for bimetallic beam with transverse connection of tubular steel of strength class K60 and titanium alloy Ti-6Al-4V

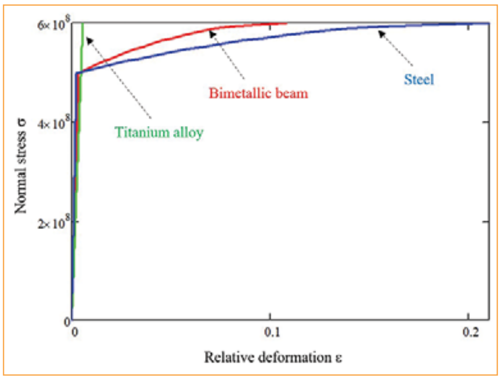

Fig. 6. Dependence of normal stresses on relative deformations for bimetallic beam with transverse connection of tubular steel of strength class K60 and titanium alloy Ti-6Al-4V

Conclusions

The new analytical inverse dependence of the relative elongations on the normal stresses $\varepsilon(\sigma)$ under stretching of a bimetallic beam with the transverse connection of metals is constructed. The inverse dependence makes it easy to numerically construct the direct dependence of normal stresses on relative elongations $\sigma(\varepsilon)$ under stretching of a bimetallic beam with the transverse connection of metals. The proposed direct and inverse analytical approximations of the metal hardening zone describe the metal hardening curve with an accuracy of 1 %. The obtained results can be widely used in metallurgy and mechanical engineering (for example, at "Vyksa Steel Works", "Chelyabinsk Tube Rolling Plant", "Zagorsky Pipe Plant", "Izhora Pipe Plant" and "Volzhsky Pipe Plant") in the production of the bimetallic sheets and large-diameter pipes.

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