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STUDY OF GEODYNAMIC PROCESSES IN MINERAL MINING USING QUASI-GEOID BASED ON WAVELET ANALYSIS

This article offers proposals on the geoid model grid output format to ensure convenient application of the data in investigation of geodynamic processes during underground mining. With a view to improving the national geoid model, it is proposed to use the data of the detail general coverage gravimetric survey of the area of the country. These data are obtained within a reasonable time using the airborn gravimetry technology.

The research findings allow creating a set of computer programs to implement the developed procedure in terms of the experimental data processing and to prove the usability of the procedure for the preliminary modeling of the geoid for the area of the Republic of Kazakhstan.

***Keywords:** geodynamic processes, physical geodesy, coordinate systems, gravitational field, geoid, quasi-geoid, gravimetric height, modeling, coordinate transformation, regulation method*
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Introduction

Global models of the Earth's gravitational field play an important role in constructing artificial Earth satellites trajectory theories and in modeling geodynamic processes and the internal Earth structure when solving various problems in the manmade and natural environment. These studies are possible by obtaining an integral picture of the processes occurring in the subsoil and on the Earth surface. Therefore, in mining, the use of a quasi-geoid is being studied in the field of rock deformations that occur during open-pit and underground mining [1, 2].

On the other hand, such height determination methods and vertical control using an accurate geoid model represent one of the most complex research topics in geodesy. Since the 1980s, this problem attracts a growing attention due to the wide spread and intensive application of the methods of the Global Navigation Satellite System (GNSS). Many of the strategies used in gravity field modeling were developed at a time when the goal of the geoid height accuracy was 10 cm or less. Currently, to determine geoid and quasi-geoid heights, it is necessary to carefully evaluate accuracy in centimeters or better, and, if necessary, adjust the approaches inherent in the methods and techniques used.

It is very important to be able to divide geodetic observations into planimetric and alimetric, and to introduce three-dimensional geodesy elements into geodetic practice at the national scale in the Republic of Kazakhstan.

In this regard, this article proposes a new technique of gravimetric quasi-geoidal (geoid) modeling for converting GPS ellipsoidal heights to normal ones, since orthometric heights do not always provide accurate results compatible with local vertical databases [3]. To improve the conversion, a gravimetric (quasi) geoidal model can be linked to GPS levelling data. The new composite surface (which is no longer a classical (quasi) geoid) can then be used to more accurately convert elevations.

Problem formulation

In the Republic of Kazakhstan, the geodetic support of critical decision-making consists of state geodetic, levelling and gravimetric networks that determine the quality and accuracy of coordinate systems, heights and gravity, including three main interconnected parts such as the state coordinate, altitude and gravimetric bases.

For many years, GPS levelling was used to validate gravimetric geoid models on land, but as geoid calculation methods improved, the data often had discrepancies (i.e., non-zero e and e′). Therefore, numerous studies focused on integration of gravimetric quasi-geoid, geoid and GPS levelling data. Many scientists used a trigonometric four-parametric surface to minimize data biases and extreme long-wavelength differences between geoid and GPS levelling data [4]. One of the most important methods is the use of parametric and non-parametric higher order surfaces such as artificial neural networks [5], spline interpolation, least squares (OLS), combined least square adjustments and other various surfaces. There are many options for defining a surface, each of the methods has its own advantages and disadvantages, describing the gravimetric (quasi) geoid combination models and GPS levelling using second-generation wavelets, and are complemented by research carried out by scientists in European countries.

Critical analysis

Currently in Kazakhstan, for (quasi) geoid measurements, the greatest practical interest lies in obtaining normal heights of points on the earth's surface from satellite surveys. The solution to the geodetic problem comes down to using a quasi-geoid (geoid) height model with the WGS–84 system processing method. In December 2002, by the decree of the Kazakhstan Government, the Baltic height system of 1977 was adopted as the state height system. The length of class 1 networks is 27 500 km, class 2 networks — 48 500 km.

Satellite-based levelling means obtaining absolute heights of points on the earth's surface based on the processing of high-precision GNSS positioning data [6]. For the satellite-based levelling implementation, it is required to have a sufficiently accurate surface model to measure absolute heights. In world practice, either a geoid surface or a quasi-geoid surface are conventionally used as such surfaces.

It is known that the geoid (from ancient Greek γῆ — Earth and other Greek εἶδος — view, literally "the exact shape of the Earth") is an even surface of potential gravity, passing through the origin for heights, or, geometrically, a closed convex surface, approximately coinciding with the water surface in the seas and oceans in a calm state, and perpendicular to the gravity direction at any point [6]. The quasi-geoid ("almost geoid") is a figure proposed in the 1950s by Soviet scientist M. S. Molodensky as a rigorous solution to the problem on the Earth's shape. Unlike the geoid, the quasi-geoid is completely determined by geodetic measurements according to Molodensky's theory [7]. Geometrically, the quasi-geoid practically coincides with the geoid in the World Ocean water and is very close to the geoid on land, deviating only by a few centimeters on flat terrain and by no more than 2 meters in mountainous areas.

The main method for creating such models was and remains the gravimetric method, the essence of which is to recalculate the measured gravity anomalies into the geoid heights using integral formulas of physical geodesy. The theoretical foundations of this method were developed by J. G. Stokes, G. Moritz, M. S. Molodensky, M. I. Yurkina, V. F. Eremeeva, V. V. Brovar and other researchers.

As noted in recent years, the relevance of geodesy and cartography, characterized by the rapid global transition to the large-scale introduction and application of satellite navigation, digital mapping and geospatial technologies, has greatly increased.

It is planned to update the state geodetic, levelling and gravimetric networks; to install a modern coordinate system using satellite technologies; to create a unified open digital geodetic platform for the entire area of the Republic, including cities and regional centers in a new coordinate system, with the information systems "State Geodetic Support" and "Basic Spatial Data". The program implementation will make it possible to launch a modern state coordinate system of Kazakhstan (Qazaqstan Terrestrial Reference System, QTRS) instead of SK-42, and to provide open spatial data on 100% of the area of the Republic. This will create conditions for integration of information systems, cadasters and geoportals of central and local government bodies.

Research methodology

In accordance with the current regulatory and technical documentation, the system of the state geodetic support in the Republic of Kazakhstan (RK), as mentioned above, presupposes availability of a state geodetic network (SGN), state levelling network (GLN) and a state gravimetric network [8]. There are various methods of the gravimetric quasi (geoid) modeling, one of which is the numerical data analysis.

In geodetic research, GPS levelling data can sometimes have errors in comparison with space research, and the classical wavelet transform associated with the Fourier theory is inapplicable to irregular data sets without pre-construction of an observation grid, therefore, the classical wavelet transform is also inapplicable. It is believed that coupling a gravimetric quasi-geoid model from GPS levelling data with second generation wavelets provides a better conversion of GPS ellipsoidal heights to normal heights.

In this regard, wavelets are a very powerful tool for analyzing numerical data widely used in image and signal processing for noise reduction, filtering and signal compression. The advantage of wavelets as against the Fourier transforms is the ability to detect both local and global frequencies. Wavelets are used in several geodetic studies and show some advantages over traditional spectral analysis methods [9].

Pre-estimation means that GPS levelling points are used to validate a combined surface that is repeated for all points in the data set. The results from the wavelet transform are compared with the calculations from the least square method (LS), while the use of the second generation wavelets may have some differences in the results (sometimes there is no need to take the deviations into account). The wavelet transform method is better suited for reducing the maximum and minimum differences between the combined geoid and GPS levelling data [10].

Interrelation between the geoid (N), ellipsoid (h) and orthometric heights (H) uses the classic expression:

$$h - H - N = e. \tag{1}$$

Similarly, the quasi-geoid (ζ), ellipsoid (h) and normal heights (H^*) are correlated by the formula:

$$h - H^*\zeta = e', \tag{2}$$

where the point height (h) is usually obtained from GPS observations; H and H^* are obtained from geodetic levelling.

The values of e and e' should be zero, but this is rarely the case due to various errors such as long-wave geoidal or quasi-geoidal errors, systematic errors in alignment networks, and geodetic errors. Thus, there are always discrepancies between the gravimetric (quasi) geoid model and the heights from GPS levelling [11].

In general, the wavelet analysis is based on two main functions: the scaling function $\varphi(x)$ and the wavelet function $\psi(x)$. The classical wavelet system contains an infinite set of translated and scaled versions of $\varphi(x)$ and $\psi(x)$ and is described by the following formula [12]:

$$\varphi_{j,k}(x) = 2^{j/2}\varphi\left(2^j x - k\right) \ \ j,k \in z. \tag{3}$$

Considering the function $f(x)$ and the mother wavelet $\psi(x)$, $f(x)$ can be expressed as a linear combination of basic functions $\psi_{j,k}(x)$ as:

$$f(x) = \sum_{j,k} a_{j,k}\psi_{j,k}(x). \tag{4}$$

Unlike the Fourier transform, which is localized only in the (global) frequency domain, the wavelet transform propagates in both spatial (or time) and frequency domains. This property introduces the wavelet analysis as a powerful tool for processing signals with spectral content variable in space (or time). Essentially, high frequency only affects the coefficient $\psi_{j,k}$ corresponding to the location at a certain geodetic point. However, classical wavelets based on the Fourier theory provide interval sampling data, while GPS levelling yields data for each observation point using the static method. In addition, wavelets are conceptually superior to least squares because they need no stationarity of observations.

The classical wavelet transform is based on the Fourier formulas [12].

It is important to emphasize that the accuracy of the obtained data allows currently studying the Earth's geometric figure dynamics, the quasi-geoid (geoid), both on a global scale and within individual regions. Considering the analysis $\{V_j\}_{j\geq 0}$ of multiple wavelength resolution in terms of the length L_2, their sequence in space is determined as:

$$V_j \subset V_{j+1} \text{ and } \bigcup_{j=0}^{\infty} V_j = L_2. \quad (5)$$

In this case, the additional wave spaces W equal to $V_{j+1} = V_j \oplus W_i$ are taken into account.

Therefore, the space V with precise resolution can be considered in wider additional spaces:

$$V_{j+1} = V_0 \oplus \bigoplus_{i=0}^{j-1} W_i. \quad (6)$$

This is called the multiscale decomposition.

Spatially, the indexes V_j and W_j are distributed within the scale $\{\psi_{j,k}\}$ according to the wavelet function $\varphi_{j,k}$. The scaling and wavelet functions are calculated from scaling functions more accurately — using some refinement coefficients h_{jlk} and g_{jlm} in the following way:

$$\varphi_{j,k} = \sum_l h_{jlk} \varphi_{j+1,k} \text{ and } \psi_{j,m} = \sum_l g_{glm} \varphi_{j+1,l} \quad (7)$$

or

$$\phi_j = \phi_{j+1} H_j \text{ and } \psi_j = \phi_{j+1} G_j, \quad (8)$$

where ϕ_j and ψ_j are the row-vectors containing the functions $\varphi_{j,k}$ and $\psi_{j,m}$, respectively; H_j and G_j are the refinement matrices.

In biorthogonal cases, decomposition and recovery of waveform use various basis functions, reconstruction involves ϕ_j and ψ_j, and their duality — $\tilde{\phi}_j$ and $\tilde{\psi}_j$. Any function $f(x)$ can now be expressed by wavelet basis functions as:

$$f(x) = \sum_k s_{j0,k} \varphi_{j0,k}(x) + \sum_{j,k} d_{j,k} \psi_{j,k}(x), \quad (9)$$

where

$$s_{j0,k} = f, \tilde{\varphi}_{j0,k} \text{ и } d_{j,k} = f, \tilde{\psi}_{j,k}. \quad (10)$$

In this case, the biorthogonal condition in the refined matrix and its dualities (i.e. filters) should take the form of:

$$\tilde{H}_j^* H_j = I, \tilde{G}_j^* H_j = 0;$$

$$\tilde{H}_j^* G_j = 0, \tilde{G}_j^* H_j = I, \quad (11)$$

where the sign * expresses the Hermitian conjugation; I is the identity matrix. Let $s_j = \{s_{j,k}\}$ and $d_j = \{d_{j,k}\}$. In this case, the direct wavelet transform is given by the formula:

$$s_j = \tilde{H}_j^* s_{j+1} \text{ и } d_j = \tilde{G}_j^* S_{j+1} \quad (12)$$

and the inverse transform — by:

$$s_{j+1} = H_j s_j + G_j d_j. \quad (13)$$

Waves subjected to the wavelet analysis can sometimes be highly correlated. This also applies to GPS levelling data where systematic distortions occur in the acquired data at the same vertical reference. Depending on the correlation value in the wave as shown in Equation 1 (values of e and e'), the wavelet coefficients are negligible, allowing the wave to be expressed by a smaller coefficient, so wavelets are useful for image compression (such as the JPEG image format). The inverse wavelet transform can be applied to threshold wavelet coefficients to reconstruct the original signal without significant (depending on the threshold) loss of information. In some cases, the resultant wave is canceled out when certain frequencies are filtered out during thresholding in the wavelet domain.

As mentioned above, the classical wavelet transform relies on the Fourier transform, so it is inapplicable to irregularly distributed data sets. However, when specifying a data grid, some (mostly high-frequency) information and geometric structure of original observations are lost. In addition, the grid is subject to smoothing; for this purpose, scientist Sideris has developed a method for implementing the Fourier transform on irregular data sets, but it is still based on a regular grid where empty cells are set to zero or include interpolated data [12].

To cope with irregular data sets, the second generation wavelets were developed by Sveldens [13]. The second generation wavelets require no transformation and single function expansion, as is the case for the classical wavelets. The second generation wavelets are built on a lifting scheme that operates in the space domain and, thus, does not rely on the Fourier transform and, therefore, avoids lattice construction. Thus, the observation geometry is preserved [14]. The lifting scheme can be used when it is impossible to apply the Fourier transform, for example, for wavelets on limited regions, wavelets on curves and surfaces, and wavelets on irregular data sets. In Equations (1) and (2), the second generation wavelets are well suited to eliminate modeling residuals [15, 16].

Considering two initial pairs of biorthogonal filters as H_j^0, G_j^0 and $\tilde{H}_j^0, \tilde{G}_j^0$, their properties can be improved using a lifting scheme. The lifting scheme states that for any operator P_j a new pair of biorthogonal filters can be found as follows:

$$\left(H_j = H_j^0 + G_j^0 P_j, G_j = G_j^0\right) \text{and} \left(\tilde{H}_j = \tilde{H}_j^0, \tilde{G}_j = \tilde{G}_j^0 - \tilde{H}_j^0 P_j\right). \quad (14)$$

Equation (14) is predictive and updates the modeling data; accordingly, it changes the role of the initial and double filters, as well as of any operator U_j:

$$(H_j = H_j^0,\ G_j = G_j^0 - H_j^0 U_j) \text{ and } \tilde{H}_j = \tilde{H}_j^0 + \tilde{G}_j^0 U_j, \tilde{G}_j = \tilde{G}_j^0. \quad (15)$$

Next, we can consider the equations for the future model:

$$e_i = m(x_i y_i) + n_i, \quad (16)$$

where e_i from Equation 1 is localized in two-dimensional space $(x_i y_i)$, and n_i – is the noise in e_i. The wavelet-type transform, adapted to irregularly distributed bivariate data, is used to estimate the function $m(x_i y_i)$. Some compactly supported scaling functions $\{\phi_j, \tilde{\phi}_j\}$ are determined at the highest level (where J is the surveillance level), and the scaling and wavelet functions at coarser levels $j = J-1, J-2$, are obtained from Equations (6) or (7) and corresponding filters [17].

Study results

The review of literature sources from foreign and domestic authors, and application practice shows that the second generation wavelet derivation can give unacceptable results; the problem is that nonequivalent transformations are unstable, i.e., they are far from orthogonal (**Fig. 1**) [18].

The second generation wavelet transform is based on a so-called lifting scheme and starts with a "lazy wavelet" that constrains the signal for even and odd choices. Odd samples are then used to predict even samples. The fragment γ_{j-1} is the predicted value subtracted from an even sample [19]. The fragments are then used to update the odd samplings to keep the mean of the signal unchanged.

Let $\lambda_{0,k} = f(x), k \in Z$ be an original signal. The first approximation based on the use of lazy wavelet is:

$$\lambda_{-1,k} = \lambda_{0,2}, k \in Z. \quad (17)$$

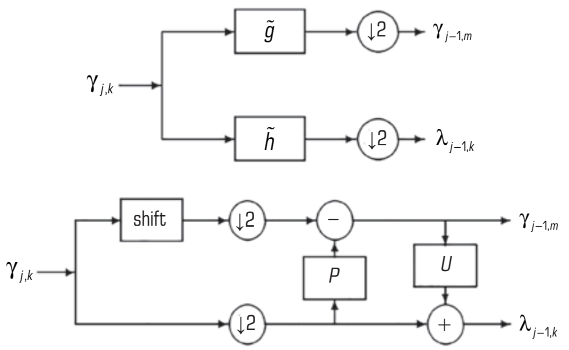

Fig. 1. Classic wavelet transform (top) and lifting scheme for the second generation wavelet transform (bottom)

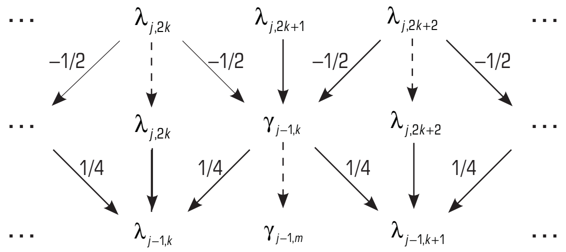

Fig. 2. Lifting diagram: calculated coefficients $\gamma_{j-1,k}$ and their use for lifting $\lambda_{j-1,k}$ in one dimension

And the wavelet coefficients are expressed as:

$$\frac{1}{2}\left(\lambda_{-1,k}+\lambda_{-1,k+1}\right),\ \ k\in Z\ . \tag{18}$$

If the signal is correlated, the wavelet coefficients ($\gamma_{i,j}$) are small and values below a certain threshold can be ignored (described below). To maintain the average approximation value (λ), approximated values ($\lambda_{-1,k}$) must be updated using wavelet fragments or coefficients. Thus, Equation (17) is modified into the following equation:

$$\tilde{\lambda}_{-1,k}=\lambda_{-1,k}+\frac{1}{4}\left(\gamma_{-1,k-1}+\gamma_{-1,k}\right), k\in Z\ . \tag{19}$$

The above calculations are presented schematically in **Fig. 2**.

A higher order scheme can be used to predict odd-indexed values in terms of even ones [20]. For example, $\lambda_{j,2k+1}$ can be predicted based on cubic polynomial interpolation in terms of the values $\lambda_{j,2k-1}$, $\lambda_{j,2k}$, $\lambda_{j,2k+2}$ and $\lambda_{j,2k+4}$. Thus, there is some interpolation built into the second generation wavelet method, but it only applies to processing of the wavelet coefficients and not to the original data.

When applying thresholds to the second generation wavelets, any coefficients less than the specified threshold are replaced by zeros [20]. Finding the optimal threshold value is an important part of smoothing or filtering [21].

The following data model (distorted by the noise n) and its wavelet transform are considered [22]:

$$y=f+n\,; \tag{20}$$

$$\omega=\tilde{W}y\,, \tag{21}$$

where $\tilde{W}$ is the direct wavelet transform, ω is the wavelet coefficients vector. The coefficients below the threshold λ are replaced by zero, and the rest remain unchanged (hard threshold) or are compressed by the value of λ (soft threshold). Applying the inverse wavelet transform to the threshold coefficients produces the filtered data:

$$y_{\lambda}=W\omega_{\lambda}\ . \tag{22}$$

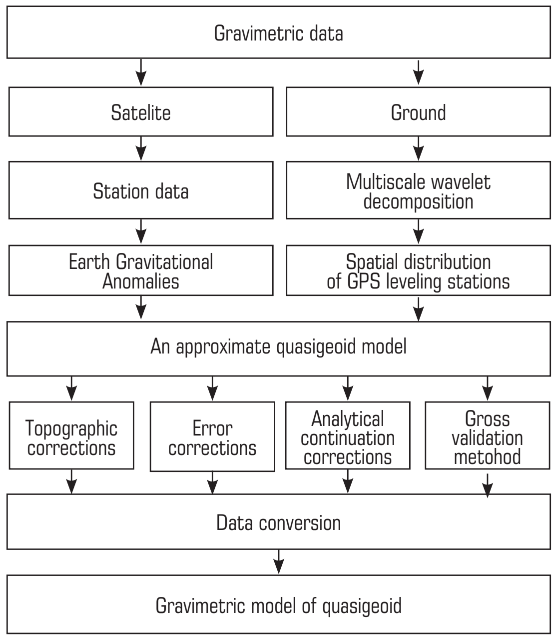

Fig. 3. Block diagram of quasi-geoid model algorithm

Wavelets must be localized in both time and frequency representation domains.

In order to apply the general methodology of using and compiling a software package and creating an algorithm for calculating and investigating the quasi-geoid model, we propose a flowchart that clearly shows the necessary levels of data processing to create a quasi-geoid model (**Fig. 3**) [23].

When designing such functions, it is inevitable to deal with the uncertainty principle, which connects the effective durations of effective functions and the width of their spectrum, with a set of different basic intentions focused on solving various problems in accordance with the presented block diagram [23].

Conclusions

To improve the accuracy of normal height determination from the local vertical coordinate system using GPS, the second generation wavelets are introduced and implemented based on the lifting scheme with a threshold factor, and on the differences between the gravimetric quasi-geoid models and GPS levelling data.

In geodynamic monitoring, this method is applied with using the models of the Earth's gravity field and relief, the data from various Earth sciences and the latest software.

For assessing geodynamic processes in the area of deformation of rocks during open pit and underground mining, the definition of spatial displacements and deformations, depth, thickness and spatial characteristics of mining are introduced.

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NUMERICAL STRESS–STRAIN MODELING OF HONEYCOMB MINE STRUCTURES WITH VERTICAL STOPES OF CYLINDRICAL FORM

The article describes the numerical modeling results of the stress–strain behavior of a room-and-pillar stoping system in honeycomb mine design including rib pillars with their angles cut off by vertical cylindrical stopes. The factors of safety are calculated for the pillars and enclosing rock mass with the excessive stresses and displacement in rock mass. The authors present a selected variant of the numerical stress–strain modeling of rib pillar with angles cut off by vertical stopes of cylindrical form in case of cellular arrangement of the stopes for the conditions of mining at the depths of 400 and 1000 m. The numerical calculation of the critical depths for using honeycomb mine structures is presented as a case-study of geological and geotechnical conditions of the Ilets rock salt deposit. The patterns of destructive loads are obtained in numerical models at different ratios of minimal widths of pillars and diameters of stopes.

Keywords: *Turner–Shevyakov hypothesis, numerical modeling, rib pillar, pillars with angles cut off by vertical stopes of cylindrical form, honeycomb mine structure, factor of safety, excessive stress, rock mass displacement, rock salt deposit*
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Introduction

The scientific works [1–7] described a new geotechnical approach and the concept of an alternative convergent geotechnology for solid mineral deposits, including the Ilets rock salt deposit [2, 3, 8, 9], based on the change of the stoping front advance, i.e., transition from horizontal stoping to top-downward or bottom-upward vertical stoping in cylindrical stopes made by drilling. Calculation of the stability of rib pillars used the Turner–Shevyakov hypothesis for the conventional room-and-pillar mining systems [10–13] and for the vertical cylinder-shaped stopes with the rib pillars with their angles cut off by circles.