In the world practice of open pit mining, drilling is mainly carried out using roller cone bits. The cost of drilling totals 25–40% of the overall cost of mining. One of the factors which worsen drilling efficiency is tracking which occurs when spiked teeth of the cones fall into the same craters cut during previous rotations of the bits, and which increases wear of the drilling tool and decreases the rate of drilling.

This article proposes a calculation algorithm for the contact paths of teeth in the peripheral rows of roller cones using actual involutes of toroidal surfaces in the periphery of the well bottom.

The algorithm provides a sufficiently accurate mesh of coverage of the well bottom periphery by tricone drill bits with the offset spin axes of roller cones with a view to preventing tracking.

The proposed algorithm can be used for the design and reasonable selection of the roller cone bits with regard to geological conditions of drilling.

**Keywords:** roller cone bit, tracking, peripheral teeth row, path, contact points, coverage mesh, tooth

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with the space between them. This leads to the increased wear of the drill bit because of the one-sided wear of its teeth, and to the drop in the rate of penetration [8, 9].

The main anti-tracking approaches are the increase in the axial force applied to the drill bit and the decrease in the space between the cutting elements in combination with the increase in their number [10]. The increase in the axial force decreases endurance of a drill bit which is a nonrecoverable object. The increase in the number of cutting elements increases the drill bit cost, the energy input and the probability of the drill teeth chippage [11, 12].

Thus, the contact path computations for the tips of teeth in the peripheral rows of tricone drill bits using involutes of actual toroidal surfaces in well bottom zones can enable proper arrangement of the teeth without changing their number, and is a promising anti-tracking method [13].

Object and Method of Study

The object of study was a roller cone bit with a diameter of 244.5 mm as schematically shown in Fig 1a. Where \( \omega_0 \) — bit rotation speed, rad/s; \( \omega_c \) — cone rotation speed, rad/s; \( R_1 \) — radius of the well, mm; \( r_a \) — radius of the cone, mm.

Tricone bits with offset axes of rolling cones form a toroidal surface in the periphery of well bottom [14–16]. In the plane of the radial section of a well, i.e. in the coordinates \( ZOY \), the cross-section of the toroidal surface can be presented as the curve \( AB \) (Fig. 1):

\[
\begin{align*}
    x &= R \sin \psi - r \sin \psi \cos \phi - r (1 - \cos \psi) \sin \psi \cos \alpha, \\
    y &= R \cos \psi + r \sin \psi \sin \phi - r (1 - \cos \psi) \cos \phi \cos \alpha, \\
    z &= r (1 - \cos \psi) \sin \psi,
\end{align*}
\]

where \( R \) is the radius of a circle along which a teeth row rolls, mm; \( r \) is the radius of the rolling row, mm; \( \alpha \) is the angle between the row plane and the plane of the well cross-section, deg; \( \phi \) is a variable parameter of the drill bit, deg.

Apparently, the curve \( AB \) has a varied radius of curvature, and it is required to find the other ways of meshing the coverage areas of toroidal surfaces in the well bottom periphery as opposed to meshing the coverage zones of spherical surfaces in operation of single roller cone bits [17].

Results

The procedure of constructing an involute curve of a toroidal loop is similar to the procedure for the involutes of spherical rings [17, 18]. It is important to know exact values of the upper limit and, in the first place, the lower limit of involutes. Heights of involutes increase one way or another during drawing of actual paths of roller cone teeth in contact with the toroidal surface, and approach the length of the curve \( AB \).

Since we know the height \( h \), and the coordinate of the point \( C \), i.e. \( OC = R_{\text{max}} = R_c \), it is easy to construct an involute of the toroidal loop (see Fig. 1) with regard to the fact that \( OA = R_1 \), and the radius of the lower limit of the involute is found from the formula:

\[
R'' = \frac{R_i}{\cos \left( \arctg \frac{h}{AR} \right)},
\]

where \( AR = (R_{\text{max}} = R_c) - R_1 > R_1 \) is the radius of the well bottom circle along which the cone nose row rolls, mm; \( R_1 \) is the well radius, mm.

The upper limit radius \( OB \) of the involute is given by:

\[
OB = \frac{R_i}{\cos \left( \arctg \frac{h}{AR} \right)},
\]

Fig. 1. Diagrams of rolling (a) and construction of involuted toroid loop of roller cone (b)

Fig. 2. Diagram of distances from lower limits of toroidal surface to points of curve \( AB \), i.e. heights \( h' \), at small increment

With the involute of the toroidal surface in the periphery of well bottom, let us discuss the algorithm of constructing contact paths of teeth edges under the above-specified conditions, i.e. with the parametric equations of the paths of these points in the Cartesian coordinates (1) and at the sufficiently accurate roller cone gear ratios related by the condition

\[
\frac{R_c}{R_0} = i = \frac{\psi_j}{\phi} \quad [19, 20].
\]

The calculation algorithm represents the successive evaluation of the following parameters:

1. The distance \( h' \) from the lower limit of the toroid to a point of the curve.

2. The path \( \psi_j' \) of the teeth row.

The set of values of \( h_j' \) (Fig. 2) is found from partition of the total height \( h = BC \) of the toroidal surface, which is a peripheral surface of the well bottom, i.e. the height of the curve \( AB \), with a small increment \( \Delta h \).

Fig. 3. Parameter \( \psi_j' \)


\[ z_i = r_j (1 - \cos \psi) \sin \alpha. \]  \hspace{1cm} (4)

and initial data from column 1 in Table 1, i.e. from the formula:

\[ \psi_j' = \arccos \left(1 - \frac{h_j^*}{r_j \sin \alpha} \right). \]  \hspace{1cm} (5)

The data set of \( \psi_j' \) is important as it leads directly to the system of parametric equations of the trajectory paths in the form of (11). The parameter is not a function of the gear ratio of the roller cone. In other words, the mode of rolling of the peripheral row is of no importance (without frictional sliding, with positive or negative sliding), and the set of \( \psi_j' \) is determinable unambiguously by placing \( h_j \) in (4) instead of \( z_j \)

\[ h_j^* = r_j (1 - \cos \psi) \sin \alpha. \]  \hspace{1cm} (6)

Herefrom we obtain the final formula:

\[ \psi_j' = \arccos \left(1 - \frac{h_j^*}{r_j \sin \alpha} \right). \]  \hspace{1cm} (7)

Naturally, the range of the set of this parameter is limited by the value of \( h_j \), i.e.:

\[ 0 \leq \psi_j' \leq \psi_j = h. \]

3. The coordinates \( x_j \) and \( y_j \) of a tooth edge point in projection onto a conditional plane XOY chosen to represent the well bottom.

The data sets of the coordinates of the tooth edge path in projection onto the plane XOY (Fig. 4), \( x_j \) and \( y_j \), are generated by calculation from the first two equations in system (1) at the certain rotation of the cone nose row by the angle \( \psi_j' \), i.e. from the formulas:

\[ x_j' = R_j \sin \phi_j - r_j \sin \psi_j' \cos (\phi_j - \gamma_j) - r_j (1 - \cos \psi_j') \sin (\phi_j - \gamma_j) \cos \alpha \]
\[ y_j' = R_j \cos \phi_j - r_j \sin \psi_j' \sin (\phi_j - \gamma_j) - r_j (1 - \cos \psi_j') \cos (\phi_j - \gamma_j) \cos \alpha \]  \hspace{1cm} (8)

when \( \phi_j = \psi_j' \) and \( \gamma = \arcsin \left( \frac{k_j}{R_j} \right) \), where \( R_j, r_j, \alpha \) and \( k \) are the geometrical parameters of the cone nose row (see Fig. 4).

\[ \Delta \rho_j' = \frac{\left( \rho_j' - \rho_j \right)^2 + \left( z_j - z_j' \right)^2}{\rho_j}. \]  \hspace{1cm} (9)

The coordinates \( x_j' \) and \( y_j' \) are the coordinates of motion of the tooth tip in the peripheral row in a new coordinate system \( X'Y'Z' \) which occurs in the same plane as XOY and has the same origin.

4. The rotational angle \( \phi_j' \) of a tooth edge point in projection onto a plane XOY chosen to represent the well bottom.

The set of data on the parameter \( \phi_j' \) gives the angles of rotation of the tooth edge point in projection onto the plane XOY. This is not the angle of rotation of the tooth edge point center round the bit axis, which is given together with the tooth edge point rotation angle \( \phi_j' \) in Fig. 4.

This parameter is calculated using the ratio

\[ \frac{x_j'}{y_j'} = \tan \phi_j'. \]  \hspace{1cm} (10)

The data set of \( \phi_j' \), alongside with the next following data set in the algorithm, is a basis to jump to the polar coordinate system. Evidently, it is impossible to transfer the curves from a toroidal surface to a plane involute at an identical accuracy.

5. The parameter \( R_j' \) which is the distance from the drill bit axis to the plane XOY.

The set of the values of \( R_j' \) is the distances from the bit axis to a projection of a test point onto the plane XOY (see Fig. 4). This parameter is given by:

\[ R_j' = \sqrt{x_j'^2 + y_j'^2}. \]  \hspace{1cm} (11)

Here, we use the Pythagoras theorem at the sufficiently accurately calculated coordinates \( x_j' \) and \( y_j' \). The data set of \( R_j' \) helps make analogies in the Cartesian and polar coordinates. This point will be explained below.

6. The parameter \( \Delta \rho_j' \) which is the distance between peaks.

The set of \( \Delta \rho_j' \) is the sum of distances between the neighbor heights \( h_j^* \) and represents a hypotenuse of rectangular triangles (Fig. 5). Generation of this data set is given analytically by:

\[ \Delta \rho_j' = \rho_j' - \rho_j. \]
\[ \Delta \rho_j' = \sqrt{(R_j' - R_i')^2 + (h_j' - h_i')^2}. \]  \hspace{1cm} (12)

\[ \Delta \rho_j' = \Delta \rho_j' + \Delta \rho_j' + \cdots + \Delta \rho_j'. \]
Starting from this stage, the algorithm generates directly the analytical structure of the polar coordinates, namely, the radius-vector; the physical sense of the latter will be explained in the next stage.

It should be emphasized that it is unadvisable to increase the values of $\Delta \rho'$ without special research. The values of $h'$ and $R'$ for generating the required data set are taken from columns 1 and 3 in Table 1. The value of $\Delta h$ in column 1 of Table 1 should be assumed to equal 1 mm for making the algorithm simpler and more accurate.

7. The parameter $\rho'_1$, which is the radius-vector in the polar coordinates.

The data set of $\rho'_1$ is the radius-vectors in the polar coordinates with the origin at the root of the involute (Fig. 6) and is generated by calculations from the formula:

$$\rho'_1 = R + \Delta \rho'_1.$$  \hspace{1cm} (13)

The parameter $R''$ is determined when constructing the involute of the toroidal surface (see Fig. 6).

Apparently, generation of the data set of the radius-vector $\rho'_1$ is perfectly accurate in constructing involutes of cones.

In this case, we sort of strengthen a concave toroidal surface and mesh the surface using craters made by tips of teeth, making it similar to a real surface.

Table 1. Parameters of roller cone No. 1

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Peripheral tooth row height — 9.843 mm; Cone nose radius — 68.000 mm; Radius of circle along which cone nose raw rolls — 121.131 mm; Cone bottom radius (R₀) = 1072.052 mm; Angle of involute at lower limit (β′₀) = 40.676 deg.

Roller cone No. 3:

1. Gear ratio — 1.391;
2. Peripheral teeth row height — 9.843 mm;
3. Cone nose radius — 68.000 mm;
4. Radius of circle along which cone nose raw rolls — 121.131 mm;
5. Cone bottom radius (R₀) = 1072.052 mm;
6. Angle of involute at lower limit (β′₀) = 40.676 deg.

The calculations show that a drill bit with the roller cones of these sizes excludes tracking as the parameters of the contact points of the peripheral row teeth (X′₀, Y′₀, ϕ′₀, β′₁) have no repeated contacts and angles at a conditional well bottom.

Conclusions

1. The coverage meshing of the involutes of the toroidal surface depends on the cutting structure of the peripheral teeth rows, as well as on the sizes and orientation of the edges of the teeth in the peripheral row relative to the planes of the cone nose rows and to the gear ratios of the roller cones.
2. The authors have proposed the calculation algorithm for the contact paths of teeth tips in the peripheral rows of tricone bits on the actual involutes of the toroidal surfaces in the periphery of well bottoms.
3. The proposed algorithm allows sufficiently accurate coverage meshing of the peripheral zones at well bottoms in tricone bit drilling at offset axes of rotation of roller cones toward anti-tracking.
4. The proposed algorithm can be used for the design and justified selection of drill bits with regard to geological conditions of drilling.